TMA 4115 Matematikk 3

Lecture for FYMA

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In this lecture we will discuss...

- Recap: Matrix algebra (adding and multiplying matrices)
- Recap: Inverse matrices
- Computing inverse matrices

Addition of matrices and multiplication with numbers is done component-wise.

For $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 42 \\ -4 & -5 & 1 \end{pmatrix}$ we have

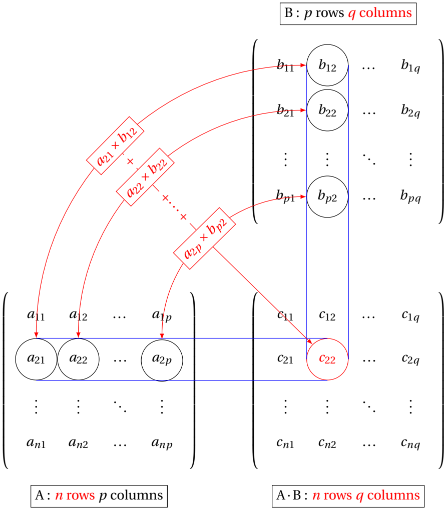
$$A + B = \begin{pmatrix} 1+1 & 2+0 & 3+42 \\ 4+(-4) & 5+(-5) & 6+1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 45 \\ 0 & 0 & 7 \end{pmatrix}$$

$$5A = \begin{pmatrix} 5*1 & 5*2 & 5*3 \\ 5*4 & 5*5 & 5*6 \end{pmatrix} = \begin{pmatrix} 5 & 10 & 15 \\ 20 & 25 & 30 \end{pmatrix}$$

Note: Addition is only defined for matrices of the same size!

Matrix multiplication is **not** done component-wise.

Matrix multiplication (diagram from Altermundus.com)



Example (similar to Høst 2008 5b)

Assume that each year 30% of owners of cars with two-wheel drive (2WD) change to a car with four-wheel drive (4WD), whilst 10% of owners of cars with 4WD change to a car with 2WD. The total number of cars is constant, and each car owner has only one car.

Question: Given that

(i) 25%, or

(ii) 10%

of car owners have 4WD now, what percentage of car owners will have 4WD in three years' time?

We will revisit examples like this later (Chapter on Markov chains).

Matrix multiplication II

Matrix multiplication behaves differently compared to multiplication of numbers.
For example, we can have $AB \neq BA$.
Another problem: $N = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq 0$ but $N^2 = 0$.

Invertible matrices

A $n \times n$ matrix M is invertible if there is a matrix B with

$$MB = I \text{ and } BM = I \quad (I \text{ identity matrix})$$

We call B the inverse of M and write $M^{-1} := B$.

Note: Only square matrices are invertible!