

TMA 4115 Matematikk 3

Lecture 2 for FYMA

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12. January 2017

Complex numbers

$a + ib$ (may also write $a + bi$) is a *complex number*

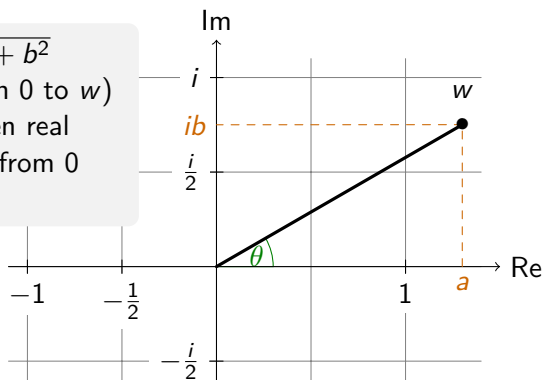
where $a, b \in \mathbb{R}$ and i the imaginary unit ($i^2 = -1$)

Representations of a complex number w :

- $a + ib$ *normal form* (or *standard form*),
here $\operatorname{Re}(w) = a$ and $\operatorname{Im}(w) = b$
- (a, b) *cartesian coordinates* for the complex plane
- (r, θ) *polar coordinates* (only for $w \neq 0$).

How to compute (r, θ) from $w = a + ib$?

$r = |w| = \sqrt{a^2 + b^2}$
(distance from 0 to w)
 θ : angle between real axis and ray from 0 through w



Recall $\arg(w) = \{\dots, \theta - 2\pi, \theta, \theta + 2\pi, \theta + 4\pi, \dots\}$

If $-\pi < \theta \leq \pi$ then $\theta = \text{Arg}(w)$ “principal argument”.

How to compute (r, θ) from $w = a + ib$, II

We know

$$r = |w| = \sqrt{a^2 + b^2}$$
$$\tan(\theta) = \tan(\arg(a + bi)) = \frac{b}{a} \quad (\text{if } a \neq 0)$$

Warning: Your calculator can compute $\tan^{-1}(\frac{b}{a})$ *but*:

$$\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{-b}{-a}\right)$$

Problem: Same number, but the angle should be different!

Solution:

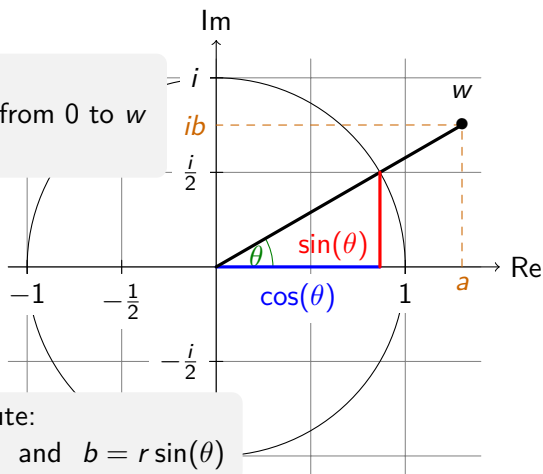
Use the two variable arctan function (called atan2)
or use \tan^{-1} and the formula for atan2 on Wikipedia
<http://en.wikipedia.org/wiki/Atan2>

Recovering $a + ib$ (or (a, b)) from (r, θ) .

We know:

r : distance from 0 to w

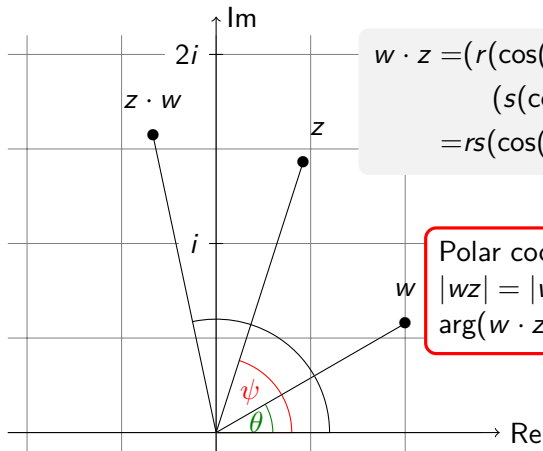
θ : angle



Then compute:

$a = r \cos(\theta)$ and $b = r \sin(\theta)$

Given $w = (r, \theta)$ and $z = (s, \psi)$ what is $w \cdot z$?



$$\begin{aligned}w \cdot z &= (r(\cos(\theta) + i \sin(\theta))) \cdot \\ &\quad (s(\cos(\psi) + i \sin(\psi))) \\ &= rs(\cos(\theta + \psi) + i \sin(\theta + \psi))\end{aligned}$$

Polar coordinates of wz

$$|wz| = |w||z| \text{ and}$$

$$\arg(w \cdot z) = \arg(w) + \arg(z)$$

Rules for the multiplication of $w, z, v \in \mathbb{C}$

$$w \cdot z = z \cdot w \text{ and } w \cdot (z \cdot v) = (w \cdot z) \cdot v \quad |w \cdot z| = |w| \cdot |z|$$