

# TMA 4115 Matematikk 3

## Lecture for FYMA

Alexander Schmeding

NTNU

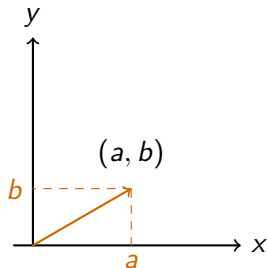
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In today's lecture we will ...

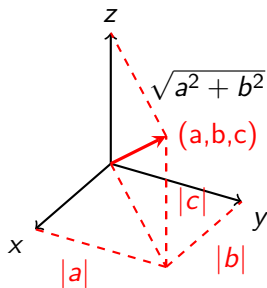
- explore the geometry of  $\mathbb{R}^n$ ,
- define the length of vectors,
- investigate when vectors are orthogonal

# Geometry of $\mathbb{R}^n$ : Length of vectors

Length in 2d:



Length in 3d:



Pythagoras:

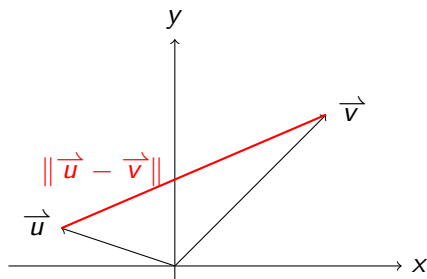
length of  $\begin{bmatrix} a \\ b \end{bmatrix}$  is  $\sqrt{a^2 + b^2}$

Pythagoras (2 times!):

length of  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  is  $\sqrt{a^2 + b^2 + c^2}$

## Distance between two vectors

How can we measure the distance between points?



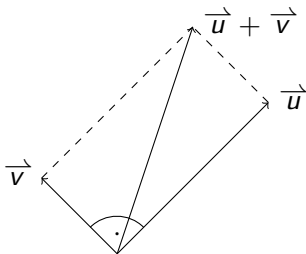
Distance between vectors

$$\text{dist}(\vec{u}, \vec{v}) = \|\vec{u} - \vec{v}\|$$

**Note:**  $\|\vec{u} - \vec{v}\| = \|\vec{v} - \vec{u}\|$ .

## What does orthogonal mean?

If vectors  $\vec{u}$  and  $\vec{v}$  in  $\mathbb{R}^2$  meet in a right angle, they are **perpendicular** (or **orthogonal**):



### Pythagoras theorem

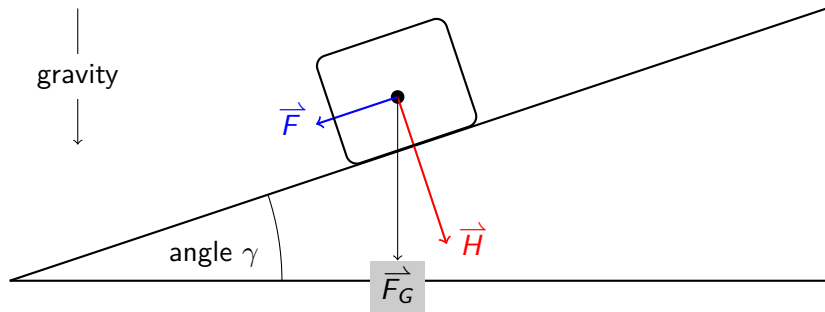
$\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2$  Comparing both sides, the equation holds if and only if:

$$\vec{u} \cdot \vec{v} = 0$$

**Idea:** Use this to define orthogonal vectors in general settings.

## Example: Splitting forces in physics

Say we know the weight of a block on a slope:



Can compute  $\vec{F}_G$  from the weight but we want:  $\vec{F}$ , the force acting on the block in the direction of the slope.

**Note:**  $\vec{F}$  and  $\vec{H}$  are orthogonal!

→ Idea: split  $\vec{F}_G$  in orthogonal components