TMA 4115 Matematikk 3

Lecture 2 for KJ & NANO

Alexander Schmeding

NTNU

12. January 2017

Complex numbers

$$a + ib \quad \text{(may also write} \quad a + bi\text{) is a } \textit{complex number}$$

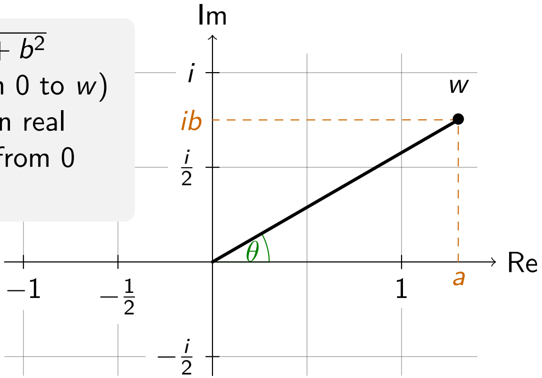
where $a, b \in \mathbb{R}$ and i the imaginary unit ($i^2 = -1$)

Representations of a complex number w:

- $a + ib$ *normal form* (or *standard form*),
  here $\mathrm{Re}(w) = a$ and $\mathrm{Im}(w) = b$
- (a, b) *cartesian coordinates* for the complex plane
- (r, θ) *polar coordinates* (only for $w \neq 0$).

How to compute (r, θ) from $w = a + ib$?

$r = |w| = \sqrt{a^2 + b^2}$
(distance from 0 to w)
θ : angle between real axis and ray from 0 through w



Recall $\arg(w) = \{\ldots, \theta - 2\pi, \theta, \theta + 2\pi, \theta + 4\pi, \ldots\}$

If $-\pi < \theta \leq \pi$ then $\theta = \operatorname{Arg}(w)$ "principal argument".

How to compute (r, θ) from $w = a + ib$, II

We know

$$r = |w| = \sqrt{a^2 + b^2}$$

$$\tan(\theta) = \tan(\arg(a + bi)) = \frac{b}{a} \qquad \text{(if } a \neq 0\text{)}$$

Warning: Your calculator can compute $\tan^{-1}(\frac{b}{a})$ *but*:

$$\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{-b}{-a}\right)$$

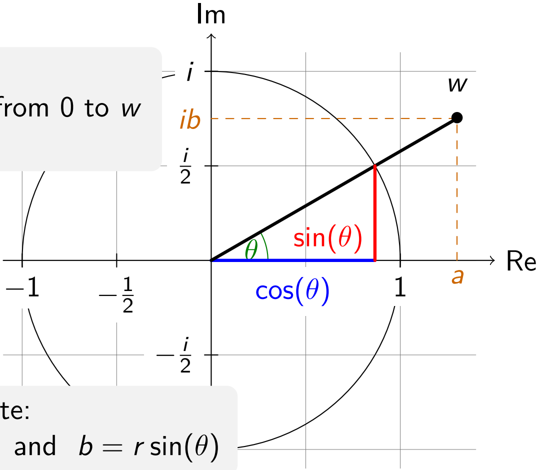
Problem: Same number, but the angle should be different!

Solution:

Use the two variable arctan function (called atan2)
or use $\tan^{-1}$ and the formula for atan2 on Wikipedia
`http://en.wikipedia.org/wiki/Atan2`

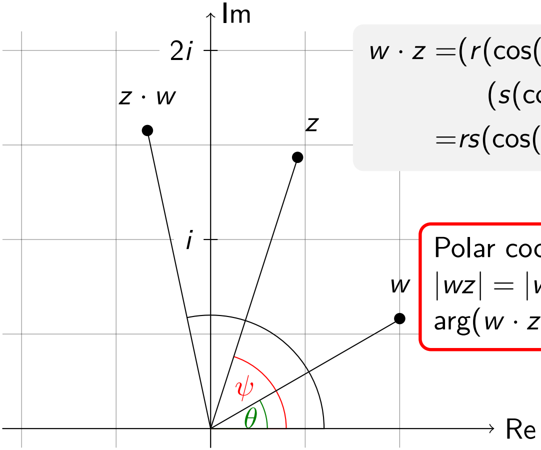
Recovering $a + ib$ (or (a, b)) from (r, θ).

We know:
r : distance from 0 to w
θ : angle



Then compute:
$a = r\cos(\theta)$ and $b = r\sin(\theta)$

Given $w = (r, \theta)$ and $z = (s, \psi)$ what is $w \cdot z$?



$$
\begin{aligned}
w \cdot z =& (r(\cos(\theta) + i\sin(\theta)))\cdot \\
& (s(\cos(\psi) + i\sin(\psi)) \\
=& rs(\cos(\theta + \psi) + i\sin(\theta + \psi))
\end{aligned}
$$

Polar coordinates of wz
$|wz| = |w||z|$ and
$\arg(w \cdot z) = \arg(w) + \arg(z)$

Rules for the multiplication of $w, z, v \in \mathbb{C}$

$w \cdot z = z \cdot w$ and $w \cdot (z \cdot v) = (w \cdot z) \cdot v$ $\quad |w \cdot z| = |w| \cdot |z|$