Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

Forkurs i kompleks analyse
Høst 2014

Øving 1

Oppgavene er hentet fra Kapittel 13 i Erwin Kreyszigs "Advanced Engineering Mathematics", 10. utgave.

1 If the product of two complex numbers is zero, show that at least one of the factors must be zero.

2 Let $z = x + iy$. Showing details, find, in terms of x and y:

a) $\mathrm{Im}(1/z)$,

b) $\mathrm{Im}(1/z^2)$.

3 Represent in polar form and graph in the complex plane. Show the details.

$$\frac{7+4i}{3-2i}.$$

4 Find and graph all the roots in the complex plane.

$$\sqrt[3]{1-i}.$$

5 Solve and graph the solutions. Show details.

$$z^2 - (6-2i)z + 17 - 6i = 0.$$

6 Prove $|\mathrm{Re}(z)| \leq |z|$, $|\mathrm{Im}(z)| \leq |z|$.

7 Determine and sketch or graph the sets in the complex plane given by:

a)

$$\frac{\pi}{2} < |z - 1 + 2i| < \pi,$$

b)

$$\mathrm{Re}\left(\frac{1}{z}\right) < 1.$$

8 Find out, and give reason, whether $f(z)$ is continuous at $z = 0$ if $f(0) = 0$ and for $z \neq 0$ the function f is equal to:

$$\frac{\mathrm{Re}(z)}{1 - |z|}.$$

9 Find the value of the derivative of $(iz^3 + 3z^2)^3$ at $2i$.

10 Show that $f(z) = \mathrm{Re}(z) = x$ is not differentiable at any z. Can you find other such functions?

11 Is the following function analytic?

$$f(z) = -\frac{i}{z^4}.$$

12 Is the following function harmonic? If your answer is yes, find a corresponding analytic function $f(z) = u(x, y) + iv(x, y)$.

$$v(x, y) = e^{-x} \sin(2y).$$

13 Determine a and b so that $u(x, y) = ax^3 + bxy$ is harmonic and find a harmonic conjugate.

14 Find all solutions and graph some of them in the complex plane.

$$e^z = 4 + 3i.$$

15 Find all solutions.

$$\sin(z) = 100.$$

16 Find $\mathrm{Ln}(z)$ when $z = 8 - 8i$.

17 Find the principal value of $(1 + i)^{1-i}$. Show details.