Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

Forkurs i kompleks analyse
Høst 2014

Øving 2

Oppgavene er hentet fra Kapittel 14 i Erwin Kreyszigs "Advanced Engineering Mathematics", 10. utgave.

1 Integrate by the first method or state why is does not apply and use the second method. Show the details.

a) C is the parabola $y = 1 + \frac{1}{2}(x-1)^2$ from $1+i$ to $3+3i$ and

$$\int_C \mathrm{Re}(z)\, dz,$$

b) C is the shortest path from $\pi/2i$ to πi and

$$\int_C e^z\, dz,$$

c) C is counterclockwise around the boundary of the square with vertices 0, i, $1+i$ and 1, and

$$\int_C \mathrm{Re}(z^2)\, dz.$$

2 For what contours C will it follow from Theorem 1 that

a)

$$\int_C \frac{dz}{1-z} = 0,$$

b)

$$\int_C \frac{\exp(1/z^2)}{z^2+4}\, dz = 0.$$

3 Integrate $f(z)$ counterclockwise around the unit circle. Indicate whether Cauchy's integral theorem applies. Show the details.

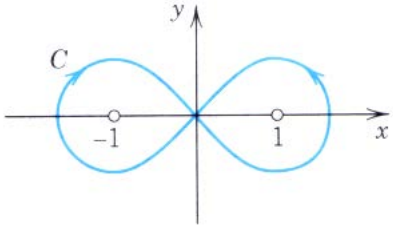
a) $f(z) = \tan(z/4)$,

b) $f(z) = 1/(\pi z - 1)$.

4 Integrate

$$\oint_C \frac{dz}{z^2-1},$$

where C is given by:



$\boxed{5}$ Integrate $z\sin(z)/(2z-1)$ around the unit circle.

$\boxed{6}$ Integrate counterclockwise or as indicated. Show the details.

a) C is given by $4x^2 + (y-2)^2 = 4$ and

$$\oint_C \frac{dz}{z^2+4},$$

b) C is given by $|z-i| = 1.4$ and

$$\oint_C \frac{\mathrm{Ln}(z+1)}{z^2+1}\,dz,$$

c) C consists of $|z| = 2$ counterclockwise and $|z| = 1$ clockwise and

$$\oint_C \frac{\exp(z^2)}{z^2(z-1-i)}\,dz.$$

$\boxed{7}$ Integrate. Show the details.

a) C consists of $|z| = 3$ counterclockwise and $|z| = 1$ clockwise and

$$\oint_C \frac{4z^3-6}{z(z-1-i)^2}\,dz,$$

b) C is $|z-3| = 2$ counterclockwise and

$$\oint_C \frac{\mathrm{Ln}(z+3)}{(z-4)^2}\,dz.$$