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Forkurs i kompleks  
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Øving 3

Oppgavene er hentet fra Kapittel 15 i Erwin Kreyszigs "Advanced Engineering Mathematics", 10. utgave.

**Hint:** I en av oppgavene kan du få bruk for at

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \approx 2.718281828.$$

**[1]** Is the given series convergent or divergent? Give a reason. Show details.

a)

$$\sum_{n=0}^{\infty} \frac{(20+30i)^n}{n!},$$

b)

$$\sum_{n=1}^{\infty} \frac{(3i)^n n!}{n^n}.$$

**[2]** Find the center and the radius of convergence.

a)

$$\sum_{n=0}^{\infty} 2^n (z-1)^n,$$

b)

$$\sum_{n=0}^{\infty} \frac{n^n}{n!} (z-\pi i)^n.$$

**[3]** Find the radius of convergence in two ways: (1) directly by the Cauchy–Hadamard formula and (2) from a series of simpler terms and termwise differentiation or integration.

a)

$$\sum_{n=2}^{\infty} \frac{n(n-1)}{4^n} (z-2i)^n,$$

b)

$$\sum_{n=1}^{\infty} \frac{2^n n(n+1)}{5^n} z^{2n}.$$

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- 4** Suppose that  $f(z)$  is even, (i.e.,  $f(-z) = f(z)$ ) and

$$f(z) = \sum_{n=0}^{\infty} a_n z^n = a_0 + a_1 z + a_2 z^2 + \dots$$

for  $|z| < R$  for some  $R > 0$ . Show that  $a_n = 0$  for odd  $n$ . Give examples.

- 5** Find the Maclaurin series and its radius of convergence:

$$\frac{z+2}{1-z^2}.$$

- 6** Find the Taylor series of  $1/z$  with center  $z_0 = i$  and its radius of convergence.

- 7** Find the radius of convergence. Try to identify the sum of the series as a familiar function.

a)

$$\sum_{n=1}^{\infty} \frac{z^n}{n},$$

b)

$$\sum_{n=0}^{\infty} \frac{z^{n+1/2}}{(2n+1)!}.$$