



Forkurs i kompleks  
analyse  
Høst 2014

Norges teknisk-naturvitenskapelige  
universitet  
Institutt for matematiske fag

Øving 4

Oppgavene er hentet fra Kapittel 16 i Erwin Kreyszigs "Advanced Engineering Mathematics", 10. utgave.

**Hint:** I en av oppgavene kan du få bruk for at

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$$

- [1] Find the Laurent series for  $e^z/(z - 1)^2$  that converges for  $0 < |z - z_0| < R$  if  $z_0 = 1$  and determine the precise region of convergence.
- [2] Find all Taylor and Laurent series with center  $z_0 = 1$  for  $1/z$ . Determine the precise regions of convergence. Show details.
- [3] Determine the location and order of the zeroes for  $\sin^4(z/2)$ .
- [4] Determine the location of the singularities, including those at infinity. For poles also state the order. Give reasons.

$$f(z) = \frac{1}{(z + 2i)^2} - \frac{z}{z - i} + \frac{z + 1}{(z - i)^2}.$$

- [5] Evaluate (counterclockwise). Show the details.
  - a)  $C$  is given by  $|z - 2 - i| = 3.2$  and
$$\oint_C \frac{z - 23}{z^2 - 4z - 5} dz,$$
  - b)  $C$  is given by  $|z - 0.2| = 0.2$  and
$$\oint_C \tan(2\pi z) dz.$$
- [6] Evaluate the following integrals and show the details of your work.

a)

$$\int_0^{2\pi} \frac{1 + \sin \theta}{3 + \cos \theta} d\theta,$$

b)

$$\int_0^{2\pi} \frac{\sin^2 \theta}{5 - 4 \cos \theta} d\theta.$$

**7** Evaluate the Cauchy principal value

$$\int_{-\infty}^{\infty} \frac{x}{8 - x^3} dx.$$

**8** Integrate  $e^{-z^2}$  around the boundary  $C$  of the rectangle with vertices  $-a, a, a + ib$  and  $-a + ib$  and let  $a \rightarrow \infty$ . Show that

$$\int_0^{\infty} e^{-x^2} \cos(2bx) dx = \frac{\sqrt{\pi}}{2} e^{-b^2}.$$