

Lecture 1: Laplace Transform

Kreyszig: Sections 6.1 and 6.2

- 1 Definition of the Laplace transform
- 2 Existence and uniqueness
- 3 Properties: Linearity, s -shift, derivatives
- 4 Many examples

Homework: Repeat partial fractions and ordinary differential equations.

Godkjenn utdanningsplanen *din i studenweb så raskt om mulig!*

Summary Lecture 1: Laplace Transform

1 $F(s) = \mathcal{L}[f](s) = \int_0^{\infty} f(t)e^{-st} dt$

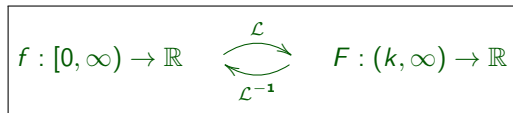
2 $L[f](s)$ exists for $s > k$ if

(A1) f is piece-wise continuous

(A2) $|f(t)| \leq Me^{kt}$ for some M and k

3 Uniqueness:

$$F(s) = G(s), s > k \Leftrightarrow f(t) = g(t), t \geq 0 \text{ (except discontin. points)}$$



Summary Lecture 1: Laplace Transform

4 Examples:

$$\mathcal{L}[\sin \omega t](s) = \frac{\omega}{s^2 + \omega^2}, \quad s > 0, \quad |\sin \omega t| \leq 1e^{0t}$$

$$\mathcal{L}[e^{at}](s) = \frac{1}{s - a}, \quad s > a, \quad |e^{at}| \leq 1e^{at}$$

5 Properties:

Linearity: $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s - a)$ for $s - a > k$

Derivatives: $\mathcal{L}[f'(t)](s) = s\mathcal{L}[f](s) - f(0)$

$$\mathcal{L}[f^{(n)}(t)](s) = s^n\mathcal{L}[f](s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$$

Popular and powerful tool to solve linear differential and integral equations