Summary: Partial differential equations

1. Concepts: Linear, homogeneous, order, solution,

Elliptic	Parabolic	Hyperbolic
Laplace $\Delta u = 0$	heat $u_t = \Delta u$	wave $u_{tt} = \Delta u$

2. Boundary value problems:

   Cauchy — u given at $t = 0$
   Dirichlet — u given on boundary
   Neumann — (normal) derivative of u given on boundary

3. Solution methods (linear problems):

Separation of variables	$u_n = F_n(x)G_n(t)$ ... superposistion	rectangular domains
Fourier transform	transform - solve - invert	whole space
D'Alembert	change of variables	1D wave equation

4. Non-homogeneous: $u = u_h + u_p$, u_h homogeneous, u_p particular solution

Lecture 13: Complex Analysis

Kreyszig: Section 13.1, 13.2, 13.3, 13.5

1. Complex numbers
2. Complex exponential function
3. Polar form
4. Roots and equations

Most of this is repetition of Matematikk 3/Lineær Algebra!!

Lecture 13: Complex numbers

$$\boxed{z = x + iy = re^{i\theta}} \quad \boxed{i^2 = -1}$$

$$x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z), \quad \bar{z} = x - iy$$

$$|z|^2 = z\bar{z} = x^2 + y^2 = r^2$$

$$r = |z|, \quad \theta = \arg(z) = \arctan(\tfrac{y}{x}) \ (\pm\pi)$$

$$\mathrm{Arg}(z) \in (-\pi, \pi]$$

Lecture 13: Complex exponential

$$\boxed{e^z = e^{x+iy} = e^x(cosy + i\sin y)}$$

$$e^{z+2\pi i} = e^z \qquad (2\pi i\text{-periodic})$$

$$e^{z_1+z_2} = e^{z_1}e^{z_2}$$

Lecture 13: Roots of complex numbers

$$w = \sqrt[n]{z}$$

$$\Leftrightarrow \quad w^n = z = re^{i\,\theta + i\,2\pi k} \quad (k \in \mathbb{Z})$$

$$\Leftrightarrow \quad w = \sqrt[n]{r}\, e^{i\,(\frac{\theta}{n} + 2\pi \frac{k}{n})}, \quad k = 0, 1, \ldots, n-1$$

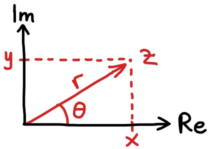
Lecture 13: Sets in $\mathbb{C}$

Circle:	$\lvert z-a \rvert = \rho$
Open disk:	$\lvert z-a \rvert < \rho$
Closed annulus:	$\rho_1 \le \lvert z-a \rvert \le \rho_2$
Half plane:	$\mathrm{Re}\, z > 0,\ \mathrm{Im}\, z \le 0, \ldots$

Complex Analysis

1. Complex number:

$$z = x + iy = re^{i\theta} \qquad \boxed{i^2 = -1}$$



2. Complex exponential function:

$$\boxed{e^z = e^{x+iy} = e^x(cosy + i\sin y)}$$

Extension of real exponential to $\mathbb{C}$

$2\pi i$-periodic: $e^{z+2\pi i} = e^z$

$e^{z_1} e^{z_2} = e^{z_1+z_2}$

3. Roots: $w = \sqrt[n]{z} \;\Leftrightarrow\; w^n = z$

$$\boxed{w = \sqrt[n]{r} e^{i(\frac{\theta}{n}+2\pi\frac{k}{n})}, \quad k = 0, 1, \ldots, n-1}$$

4. Sets:

Circle:	$\lvert z-a\rvert = \rho$
Open disk:	$\lvert z-a\rvert < \rho$
Closed annulus:	$\rho_1 \leq \lvert z-a\rvert \leq \rho_2$
Half plane:	$\mathrm{Re}\, z > 0$, $\mathrm{Im}\, z \leq 0, \ldots$