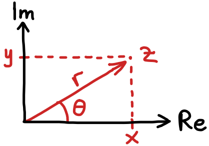
Complex Analysis



1. Complex number:

   $z = x + iy = re^{i\theta}$ $\qquad$ $\boxed{i^2 = -1}$

2. Complex exponential function:

   $$\boxed{e^z = e^{x+iy} = e^x(cosy + i\sin y)}$$

   Extension of real exponential to $\mathbb{C}$

   $2\pi i$-periodic: $e^{z+2\pi i} = e^z$

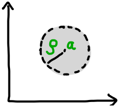
   $e^{z_1} e^{z_2} = e^{z_1+z_2}$

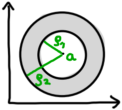
3. Roots: $w = \sqrt[n]{z} \;\Leftrightarrow\; w^n = z$

   $$\boxed{w = \sqrt[n]{r} e^{i(\frac{\theta}{n} + 2\pi\frac{k}{n})}, \quad k = 0, 1, \ldots, n-1}$$

4. Sets:

Circle:	$\lvert z - a\rvert = \rho$
Open disk:	$\lvert z - a\rvert < \rho$
Closed annulus:	$\rho_1 \le \lvert z - a\rvert \le \rho_2$
Half plane:	$\mathrm{Re}\, z > 0$, $\mathrm{Im}\, z \le 0$, ...





Lecture 14: Complex Analysis

Kreyszig: Section 13.3, 13.4

1. Sets: Open, connected, domains
2. Complex functions
3. Limits, continuity, derivative
4. Analytic functions, Cauchy-Riemann equations

Sets – as in $\mathbb{R}^2$

Limits, continuity – as for functions $f : \mathbb{R}^2 \to \mathbb{R}^2$

Derivatives – as for functions $f : \mathbb{R} \to \mathbb{R}$

OBS: Du trenger 8 (av 12) øvinger godkjent for å få ta eksamen!!

Open (contains neighborhood of each point)

Closed (complement open)

Connected (a curve connects any two points)

Domain (open, connected)

Lecture 14: Complex functions

A function f

a rule assigning each $z \in S$ a unique value $f(z) \in \mathbb{C}$

S: domain of definition

$w = f(z) = f(x + iy) = u(x, y) + iv(x, y)$

Limits, continuity (as for function $\mathbb{R}^2 \to \mathbb{R}^2$)

Derivatives ($\approx$ as for functions $\mathbb{R} \to \mathbb{R}$)

- differentiation rules as for real functions

$f(z)$ analytic in domain D

if f defined and *differentiable* for all $z \in D$

$$\Updownarrow$$

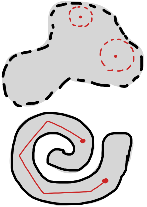
Cauchy-Riemann equations hold in D:

$$\boxed{u_x = v_y, \quad u_y = -v_x}$$

Summary: Complex Analysis

1. Sets in $\mathbb{C}$:

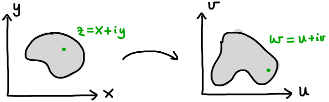
   Open: Contains open disk about each point
   Connected: Any two points can be connected
   by a finite continuous curve within the set
   Domain: Open and connected



2. Complex functions:

   Assigns each z in the domain of definition a unique value $f(z) \in \mathbb{C}$

   $\boxed{w = f(z) = u(x, y) + iv(x, y)}$ $\boxed{u = \mathrm{Re}\, w, \quad v = \mathrm{Im}\, w}$



3. Limit, continuity: Same as for functions of 2 real variables

4. Derivative: Same definition/rules as for functions of one real variable

5. Analytic functions:

   $f(z)$ analytic in domain D if defined and *differentiable* in all $z \in D$

   $\Leftrightarrow$ Cauchy-Riemann equations hold in D: $\boxed{u_x = v_y, \quad u_y = -v_x}$