Summary: Complex Analysis

1. Analytic functions and Cauchy-Riemann equations:

   $f(z) = u(x,y) + iv(x,y)$ analytic in domain D

   $$\Updownarrow$$

   u_x, u_y, v_x, v_y exists, are continuous, and

   $$\boxed{u_x = v_y \quad \text{and} \quad u_y = -v_x} \qquad \text{in} \qquad D.$$

2. Laplace equation $u_{xx} + u_{yy} = 0$

   $f(z) = u(x,y) + iv(x,y)$ analytic in domain D

   $$\Downarrow$$

   u, v are 2 times continuously differentiable, and

   $$\boxed{u_{xx} + u_{yy} = 0 \quad \text{and} \quad v_{xx} + v_{yy} = 0} \qquad \text{in} \qquad D.$$

   u, v are conjugate harmonic functions.

3. Conformal mappings:

   Maps preserving angles and orientation between smooth curves

   $$\boxed{f \text{ analytic in } D \quad \Rightarrow \quad f \text{ conformal where } f' \neq 0 \text{ in } D}$$

   $f(z) = z^n$ conformal at $z \neq 0$, $\quad z = 0$: $\arg(\dot{w}_1 - \dot{w}_2) = n \arg(\dot{z}_1 - \dot{z}_2)$

Lecture 16: Complex Analysis

Kreyszig: Sections 13.5, 13.6, 13.7

1. Expenonential function
2. Trigonometic and hyperbolic functions
3. Logarithm

OBS: Du trenger 8 (av 12) øvinger godkjent for å få ta eksamen!!

Lecture 16: Exponential function

$$\boxed{e^z = e^{x+iy} = e^x(\cos y + i \sin y)} \qquad 2\pi i\text{-periodic}$$

$$|e^z| = e^x, \quad |e^{iy}| = 1, \quad \arg e^z = y + 2\pi n,\ n \in \mathbb{Z}$$

$$(e^z)' = e^z \quad (\text{analytic in } \mathbb{C})$$

$$e^z \neq 0 \text{ in } \mathbb{C} \quad (\text{conformal in } \mathbb{C})$$

Lecture 16: Trigonometric/hyperbolic functions

$$\begin{array}{llr}
\cos z = \frac{1}{2}(e^{iz} + e^{-iz}) & \sin z = \frac{1}{2i}(e^{iz} - e^{-iz}) & (p = 2\pi) \\
\cosh z = \frac{1}{2}(e^{z} + e^{-z}) & \sinh z = \frac{1}{2}(e^{z} - e^{-z}) & (p = 2\pi i) \\
\tan z = \frac{\sin z}{\cos z} & \ldots &
\end{array}$$

$$\cos^2 z + \sin^2 z = 1 \qquad \text{and} \qquad \cosh^2 z - \sinh^2 z = 1$$

$(\cos z)' = -\sin z, \ \ldots$ same as for real functions

Lecture 16: Logarithm

Definition: $w = \ln z \quad \Longleftrightarrow \quad e^w = z$

$$\ln z = \ln|z| + i(\mathrm{Arg}\, z + 2\pi n), \quad n \in \mathbb{Z}$$

$$\mathrm{Ln}\, x = \ln|z| + i\,\mathrm{Arg}\, z \quad \text{(principal value)}$$

Summary: Complex Analysis

1. Exponential function:

   $$\boxed{e^z = e^{x+iy} = e^x(\cos y + i \sin y)} \qquad 2\pi i\text{-periodic}$$

   $|e^z| = e^x, \qquad |e^{iy}| = 1, \qquad \arg e^z = y + 2\pi n,\ n \in \mathbb{Z}$

   $e^{z_1} e^{z_2} = e^{z_1 + z_2}$

   $(e^z)' = e^z$ (analytic in $\mathbb{C}$), $\quad e^z \neq 0$ in $\mathbb{C}$ (conformal in $\mathbb{C}$)

2. Trigonometric and hyperbolic functions:

   $$\boxed{\cos z = \tfrac{1}{2}(e^{iz} + e^{-iz})} \quad \boxed{\sin z = \tfrac{1}{2i}(e^{iz} - e^{-iz})} \qquad 2\pi \text{ periodic}$$

   $$\boxed{\cosh z = \tfrac{1}{2}(e^{z} + e^{-z})} \quad \boxed{\sinh z = \tfrac{1}{2}(e^{z} - e^{-z})} \qquad 2\pi i \text{ periodic}$$

   $$\boxed{\tan z = \frac{\sin z}{\cos z}} \qquad \ldots$$

   $\cos^2 z + \sin^2 z = 1 \quad$ and $\quad \cosh^2 z - \sinh^2 z = 1$

   $(\cos z)' = -\sin z, \ldots$ derivation as for real functions

3. Logarithm:

   $$\boxed{\ln z = \ln|z| + i(\mathrm{Arg}\, z + 2\pi n), \quad n \in \mathbb{Z}}$$

   $\mathrm{Ln}\, z = \ln|z| + i\,\mathrm{Arg}\, z$ (principal value)