Summary: Complex Analysis

1. Exponential function:

$$e^z = e^{x+iy} = e^x(\cos y + i\sin y)$$ $2\pi i$-periodic

$|e^z| = e^x$, $|e^{iy}| = 1$, $\arg e^z = y + 2\pi n,\ n \in \mathbb{Z}$

$e^{z_1} e^{z_2} = e^{z_1+z_2}$

$(e^z)' = e^z$ (analytic in $\mathbb{C}$), $e^z \neq 0$ in $\mathbb{C}$ (conformal in $\mathbb{C}$)

2. Trigonometric and hyperbolic functions:

$$\cos z = \frac{1}{2}(e^{iz} + e^{-iz}) \qquad \sin z = \frac{1}{2i}(e^{iz} - e^{-iz})$$ 2π periodic

$$\cosh z = \frac{1}{2}(e^z + e^{-z}) \qquad \sinh z = \frac{1}{2}(e^z - e^{-z})$$ $2\pi i$ periodic

$$\tan z = \frac{\sin z}{\cos z} \qquad \ldots$$

$\cos^2 z + \sin^2 z = 1$ and $\cosh^2 z - \sinh^2 z = 1$

$(\cos z)' = -\sin z, \ldots$ derivation as for real functions

3. Logarithm:

$$\ln z = \ln|z| + i(\mathrm{Arg}\, z + 2\pi n), \quad n \in \mathbb{Z}$$

$\mathrm{Ln}\, z = \ln|z| + i\,\mathrm{Arg}\, z$ (principal value)

Lecture 17: Complex Analysis

Kreyszig: Sections 13.7, 14.1

1. Logarithm
2. Complex Line integral
3. Examples

OBS: Du trenger 8 (av 12) øvinger godkjent for å få ta eksamen!!

Lecture 17: Logarithm and general powers

$$\ln x = \ln |z| + i(\mathrm{Arg}\, z + 2\pi n), \qquad n \in \mathbb{Z}$$

$$\mathrm{Ln}\, x = \ln |z| + i\,\mathrm{Arg}\, z \quad \text{(principal value)}$$

$\boxed{\mathrm{Ln}\, z \text{ analytic}}$ except at $z = 0$ and negativ real axis

$$\boxed{(\mathrm{Ln}\, z)' = \frac{1}{z}}$$

$$z^c \overset{\mathrm{DEF}}{=} e^{c \ln z}$$

Lecture 17: Complex line integral

Defined via Riemann-sums:

$$\int_C f(z)dz = \lim_{n\to\infty, |P_n|\to 0} \sum_{k=1}^{n} f(\tilde{z}_k)\Delta z_k$$

Exists and uniquely defined when:

(A1) C piecewise smooth, oriented, finite length

(A2) f is continuous on C

$C : z(t), t \in [a, b]$:

$$\boxed{\int_C f(z)dz = \int_a^b f(z(t))\dot{z}(t)dt}$$

Lecture 17: Complex line integral

$$\int_C [af(z) + bg(z)]dz = a\int_C f(z)dz + b\int_C g(z)dz$$

$$\int_{-C} f(z)dz = -\int_C f(z)dz$$

$$\int_{C_1 \cup C_2} f(z)dz = \int_{C_1} f(z)dz + \int_{C_2} f(z)dz, \qquad C_1 \cap C_2 = \emptyset$$

Summary: Complex Analysis

1. Logarithm:

   $$\boxed{\ln x = \ln|z| + i(\mathrm{Arg}\, z + 2\pi n), \qquad n \in \mathbb{Z}}$$

   $\mathrm{Ln}\, x = \ln|z| + i\mathrm{Arg}\, z$ (principal value)

   $\boxed{(\mathrm{Ln}\, z)' = \frac{1}{z}}$ $\boxed{\mathrm{Ln}\, z \text{ analytic}}$ except at $z = 0$ and negativ real axis

   $z^c \overset{\mathrm{DEF}}{=} e^{c \ln z}$

2. Complex line integral:

   Defined via Riemann-sums, exists and uniquely defined when:

   (A1) C piecewise smooth, oriented curve with finite length

   (A2) f is continuous on C

   $\boxed{\int_C f(z)dz = \int_a^b f(z(t))\dot{z}(t)dt}$ when $C : z(t), t \in [a, b]$

   $\int_C [af(z) + bg(z)]dz = a\int_C f(z)dz + b\int_C g(z)dz$

   $\int_{-C} f(z)dz = -\int_C f(z)dz$

   $\int_{C_1 \cup C_2} f(z)dz = \int_{C_1} f(z)dz + \int_{C_2} f(z)dz$ when $C_1 \cap C_2 = \emptyset$