

Summary: Laplace Transform

① $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$

② $L[f](s)$ exists for $s > k$ if

(A1) f is piece-wise continuous

(A2) $|f(t)| \leq Me^{kt}$ for some M and k

③ Uniqueness:

$$F(s) = G(s), s > k \quad \Leftrightarrow \quad f(t) = g(t), t \geq 0 \text{ (except discontin. points)}$$

$$\boxed{f : [0, \infty) \rightarrow \mathbb{R} \quad \begin{array}{c} \xrightarrow{\mathcal{L}} \\ \xleftarrow{\mathcal{L}^{-1}} \end{array} \quad F : (k, \infty) \rightarrow \mathbb{R}}$$

Summary: Laplace Transform

4 Examples:

$$\mathcal{L}[\sin \omega t](s) = \frac{\omega}{s^2 + \omega^2}, \quad s > 0, \quad |\sin \omega t| \leq 1e^{0t}$$

$$\mathcal{L}[e^{at}](s) = \frac{1}{s - a}, \quad s > a, \quad |e^{at}| \leq 1e^{at}$$

5 Properties:

Linearity: $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s - a)$ for $s - a > k$

Derivatives: $\mathcal{L}[f'(t)](s) = s\mathcal{L}[f](s) - f(0)$

$$\mathcal{L}[f^{(n)}(t)](s) = s^n \mathcal{L}[f](s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$$

Popular and powerful tool to solve linear differential and integral equations

Lecture 2: Laplace Transform

Kreyszig: Sections 6.2 and 6.3

- 1 Laplace transform of an integral
- 2 Solving differential equations with the Laplace transform
- 3 t-shifting
- 4 Heaviside functions
- 5 Many examples

Homework: Repeat partial fractions and ordinary differential equations.

Godkjenn utdanningsplanen *din i studenweb så raskt om mulig!*

Summary: Laplace Transform

1 $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$

2 $f : [0, \infty) \rightarrow \mathbb{R} \quad \begin{array}{c} \xrightarrow{\mathcal{L}} \\ \xleftarrow{\mathcal{L}^{-1}} \end{array} \quad F : (k, \infty) \rightarrow \mathbb{R}$

3 Unit step function:

$$u(t-s) = \begin{cases} 0, & t < a \\ 1, & t > 0 \end{cases}$$

$$\mathcal{L}[u(t-a)](s) = \frac{1}{s}e^{-as}$$

Summary: Laplace Transform

4 Properties:

Linearity: $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s - a)$ for $s - a > k$

t-shift: $\mathcal{L}[f(t - a)u(t - a)](s) = e^{-as}\mathcal{L}[f](s)$

Derivatives: $\mathcal{L}[f'(t)](s) = s\mathcal{L}[f](s) - f(0)$

$$\mathcal{L}[f''(t)](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$

Integral: $\mathcal{L}[\int_0^t f(\tau)d\tau](s) = \frac{1}{s}\mathcal{L}[f](s)$

Summary: Laplace Transform

5 Solving equations with the Laplace transform:

