Bestem $I = \int_0^\infty \frac{\cos x - \cos 5x}{2x^2}\,dx$

(int. eks.)

$2 \cdot \pi R$

Obs: $I \overset{\text{like integrand}}{=} \frac{1}{2}\int_{-\infty}^{\infty} \dots \, dx$

$$4I \overset{\text{s.ik.}}{=} \mathrm{Re}\Big\{ \underbrace{\text{pr.v.}\int_{-\infty}^{\infty} \underbrace{\frac{e^{ix} - e^{i5x}}{x^2}}_{\overset{a)}{=} f(x)}\,dx}_{\tilde{I}} \Big\}$$

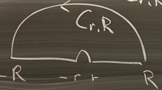
a) $\Rightarrow$ eneste sing for

f = orden 1 pol i $z = 0$

1) $\hat{I} = \lim_{\substack{r \to 0 \\ R \to \infty}} \hat{I}_{r,R}$

$$\hat{I}_{r,R} = \left(\int_{-R}^{-r} + \int_{r}^{R} \right) f(x)\,dx$$

2) $\tilde{I}_{r,R} = \left(\oint_{C_{r,R}} - \int_{S_R} - \int_{-S_r}\right) f(z)\,dz$



3) $\oint_{C_{r,R}} f\,dz = 0$

4) $\left|\int_{S_R} f\,dz\right| \xrightarrow[R\to\infty]{b)} 0$

5) $\int_{-S_r} f\,dz = -\int_{S_r} f\,dz \xrightarrow[r\to 0]{a)} \underline{4\pi}$

Konklusjon:

$$I = \frac{1}{4}\,\mathrm{Re}\,\hat{I} = \frac{1}{4}\left(0-0-(-4\pi)\right) = \underline{\pi}$$