Lecture 1: Laplace Transform

Kreyszig: Sections 6.1 and 6.2

1. Defintion of the Laplace transform
2. Existence and uniqueness
3. Properties: Linearity, s-shift, derivatives
4. Many examples

Homework: Repeat partial fractions and ordinary differential equations.

Godkjenn utdanningsplanen *din i studenweb før torsdag 25.08.!*

Summary Lecture 1: Laplace Transform

1. $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$

2. $L[f](s)$ exists for $s > k$ if

   (A1) f is piece-wise continuous

   (A2) $|f(t)| \leq Me^{kt}$ for some M and k

3. Uniqueness:

   $F(s) = G(s),\ s > k \quad \Leftrightarrow \quad f(t) = g(t),\ \ t \geq 0$ (except discont. points)

$$f : [0, \infty) \to \mathbb{R} \quad \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\rightleftarrows}} \quad F : (k, \infty) \to \mathbb{R}$$

Summary Lecture 1: Laplace Transform

4. Examples:

$$\mathcal{L}[\sin \omega t](s) = \frac{\omega}{s^2+\omega^2},\ s>0, \qquad |\sin \omega t| \leq 1e^{0t}$$

$$\mathcal{L}[e^{at}](s) = \frac{1}{s-a},\ s>a, \qquad |e^{at}| \leq 1e^{at}$$

5. Properties:

Linearity: $\mathcal{L}[af(t)+bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s-a)$ for $s-a>k$

Derivatives: $\mathcal{L}[f'(t)](s) = s\mathcal{L}[f](s) - f(0)$

$$\mathcal{L}[f^{(n)}(t)](s) = s^n\mathcal{L}[f](s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$$

Popular and powerful tool to solve linear differential and integral equations