

Summary: Partial differential equations

- ① **Concepts:** Linear, homogeneous, order, solution,

Elliptic	Parabolic	Hyperbolic
Laplace $\Delta u = 0$	heat $u_t = \Delta u$	wave $u_{tt} = \Delta u$

- ② **Boundary value problems:**

Cauchy u given at $t = 0$

Dirichlet u given on boundary

Neumann (normal) derivative of u given on boundary

- ③ **Solution methods (linear problems):**

Separation of variables	$u_n = F_n(x)G_n(t)$... superposition	rectangular domains
Fourier transform	transform - solve - invert	whole space
D'Alembert	change of variables	1D wave equation

- ④ **Non-homogeneous:** $u = u_h + u_p$, u_h homogeneous, u_p particular solution

Lecture 13: Complex Analysis

Kreyszig: Section 13.1, 13.2, 13.3, 13.5

- 1 Complex numbers
- 2 Complex exponential function
- 3 Polar form
- 4 Roots and equations

Most of this is repetition of Matematikk 3!!

Lecture 13: Complex numbers

$$z = x + iy = (x, y) = re^{i\theta}$$

$$i^2 = -1$$

$$x = \operatorname{Re}(z), \quad y = \operatorname{Im}(z), \quad \bar{z} = x - iy$$

$$|z|^2 = z\bar{z} = x^2 + y^2 = r^2$$

$$r = |z|, \quad \theta = \arg(z) = \arctan\left(\frac{y}{x}\right) (\pm\pi)$$

$$\operatorname{Arg}(z) \in (-\pi, \pi]$$

Lecture 13: Complex exponential

$$e^z = e^{x+iy} = e^x(\cos y + i \sin y)$$

$$e^{z+2\pi i} = e^z \quad (2\pi i\text{-periodic})$$

$$e^{z_1+z_2} = e^{z_1}e^{z_2}$$

Lecture 13: Roots of complex numbers

$$w = \sqrt[n]{z}$$

$$\Leftrightarrow w^n = z ; \quad w = Re^{i\varphi}, \quad z = re^{i\theta}$$

$$\Leftrightarrow R^n e^{in\varphi} = re^{i\theta + i2\pi k} \quad (k \in \mathbb{Z})$$

$$\Leftrightarrow w = \sqrt[n]{r} e^{i(\frac{\theta}{n} + 2\pi \frac{k}{n})}, \quad k = 0, 1, \dots, n-1$$

Lecture 13: Sets in $\mathbb{C}$

Circle: $|z - a| = \rho$

Open disk: $|z - a| < \rho$

Closed annulus: $\rho_1 \leq |z - a| \leq \rho_2$

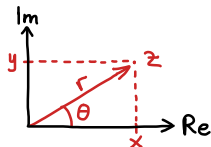
Half plane: $\operatorname{Re} z > 0, \operatorname{Im} z \leq 0, \dots$

Summary: Complex Analysis

1 Complex number:

$$z = x + iy = (x, y) = re^{i\theta}$$

$$i^2 = -1$$



2 Complex exponential function:

$$e^z = e^{x+iy} = e^x(\cos y + i \sin y)$$

Extension of real exponential to $\mathbb{C}$

$$2\pi i\text{-periodic: } e^{z+2\pi i} = e^z$$

$$e^{z_1} e^{z_2} = e^{z_1+z_2}$$

3 Roots: $w = \sqrt[n]{z} \Leftrightarrow w^n = z$

$$w = \sqrt[n]{r} e^{i(\frac{\theta}{n} + 2\pi \frac{k}{n})}, \quad k = 0, 1, \dots, n-1$$

4 Sets:

Circle: $|z - a| = \rho$

Open disk: $|z - a| < \rho$

Closed annulus: $\rho_1 \leq |z - a| \leq \rho_2$

Half plane: $\operatorname{Re} z > 0, \operatorname{Im} z \leq 0, \dots$

