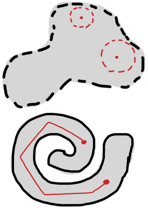
Complex Analysis

1. Sets in $\mathbb{C}$:

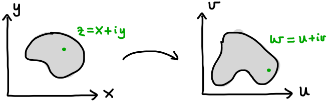
   Open: Contains open ball about each point
   Connected: Any two points can be connected by a finite continuous curve within the set
   Domain: Open and connected



2. Complex functions:

   Assigns each z in the domain of definition a unique value $f(z) \in \mathbb{C}$

   $$w = f(z) = u(x, y) + iv(x, y) \qquad u = \mathrm{Re}\, w, \quad v = \mathrm{Im}\, w$$



3. Limit, continuity: Same as for functions of 2 real variables
4. Derivative: Same definition/rules as for functions of one real variable
5. Analytic functions:

   $f(z)$ analytic in domain D if defined and *differentiable* in all $z \in D$

   $\Leftrightarrow$ Cauchy-Riemann equations hold in D: $u_x = v_y, \quad u_y = -v_x$

Lecture 15: Complex Analysis

Kreyszig: Sections 13.4, 17.1

1. Cauchy-Riemann equations
2. Laplace equation, Harmonic functions
3. Conformal mappings

OBS: Du trenger 8 (av 12) øvinger godkjent for å få ta eksamen!!

Lecture 15: The Cauchy-Riemann equations

(CR) $\boxed{u_x = v_y}$ and $\boxed{u_y = -v_x}$

$f(z) = u(x,y) + iv(x,y)$ analytic in domain D

$$\Updownarrow$$

u_x, u_y, v_x, v_y exists, continuous, and satisfy (CR) in D

(Laplace) $$u_{xx} + u_{yy} = 0$$

$f(z) = u(x,y) + iv(x,y)$ analytic in domain D

$$\Downarrow$$

u, v are $2\times$cont. differentiable + satisfy (Laplace) in D

$\rightarrow$ u, v are conjugate harmonic functions

Preserves angles and orientation between smooth curves

$$\boxed{f \text{ analytic in } D \quad \Rightarrow \quad f \text{ conformal where } f' \neq 0 \text{ in } D}$$

Ex: $f(z) = z^n$ conformal at $z \neq 0$

At $z = 0$: $\arg(\dot{w}_1 - \dot{w}_2) = n \arg(\dot{z}_1 - \dot{z}_2)$

Summary: Complex Analysis

1. Analytic functions and Cauchy-Riemann equations:

   $f(z) = u(x,y) + iv(x,y)$ analytic in domain D

   $\Updownarrow$

   u_x, u_y, v_x, v_y exists, are continuous, and

   $$\boxed{u_x = v_y \quad \text{and} \quad u_y = -v_x} \qquad \text{in} \qquad D.$$

2. Laplace equation $u_{xx} + u_{yy} = 0$

   $f(z) = u(x,y) + iv(x,y)$ analytic in domain D

   $\Downarrow$

   u, v are 2 times continuously differentiable, and

   $$\boxed{u_{xx} + u_{yy} = 0 \quad \text{and} \quad v_{xx} + v_{yy} = 0} \qquad \text{in} \qquad D.$$

   u, v are conjugate harmonic functions.

3. Conformal mappings:

   Maps preserving angles and orientation between smooth curves

   $$\boxed{f \text{ analytic in } D \quad \Rightarrow \quad f \text{ conformal where } f' \neq 0 \text{ in } D}$$

   $f(z) = z^n$ conformal at $z \neq 0$, $\quad z = 0$: $\arg(\dot{w}_1 - \dot{w}_2) = n \arg(\dot{z}_1 - \dot{z}_2)$