Summary Lecture 17: Complex Analysis

1. Logarithm:

   $$\boxed{\ln x = \ln|z| + i(\mathrm{Arg}\, z + 2\pi n), \qquad n \in \mathbb{Z}}$$

   $\mathrm{Ln}\, x = \ln|z| + i\mathrm{Arg}\, z$ (principal value)

   $\boxed{(\mathrm{Ln}\, z)' = \frac{1}{z}}$ $\boxed{\mathrm{Ln}\, z \text{ analytic}}$ except at $z = 0$ and negativ real axis

   $z^c \overset{\mathrm{DEF}}{=} e^{c \ln z}$

2. Complex line integral:

   Defined via Riemann-sums, exists and uniquely defined when:

   (A1) C piecewise smooth, oriented curve with finite length

   (A2) f is continuous on C

   $\boxed{\int_C f(z)dz = \int_a^b f(z(t))\dot{z}(t)dt}$ when $C : z(t), t \in [a, b]$

   $\int_C [af(z) + bg(z)]dz = a\int_C f(z)dz + b\int_C g(z)dz$

   $\int_{-C} f(z)dz = -\int_C f(z)dz$

   $\int_{C_1 \cup C_2} f(z)dz = \int_{C_1} f(z)dz + \int_{C_2} f(z)dz$ when $C_1 \cap C_2 = \emptyset$

Lecture 18: Complex Analysis

Kreyszig: Sections 14.1, 14.2

1. Complex Line integral
2. Cauchy integral theorem
3. Independence of path
4. The indefinite integral
5. Domains with holes
6. Examples

Lecture 18: Complex line integral

$$\int_C [af(z) + bg(z)]dz = a\int_C f(z)dz + b\int_C g(z)dz$$

$$\int_{-C} f(z)dz = -\int_C f(z)dz$$

$$\int_{C_1 \cup C_2} f(z)dz = \int_{C_1} f(z)dz + \int_{C_2} f(z)dz \text{ when } C_1 \cap C_2 = \emptyset$$

$$\boxed{\left|\int_C f(z)dz\right| \leq M \cdot L}$$

$$\text{where} \quad M = \max_{z \in C} |f(z)|, \quad L = \text{length}(C)$$

Lecture 18: Cauchy's integral theorem

f analytic in simply connected domain D, $C \subset D$ simple, closed curve

$$\implies \quad \oint_C f(z)dz = 0$$

Consequences:

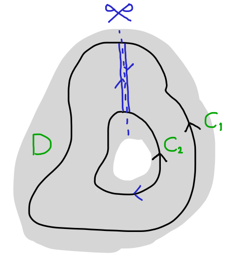
(a) $\int_C f(z)dz$ is independent of path in D

(depends on endpoints of C only)

(b) The indefinite integral of f exists in D

Cut to have a simply connected domain...

... add segments along cut to have closed curve...



Cauchy in the cut domain D^*:

$$\oint_{C_1} f(z)dz = \oint_{C_2} f(z)dz$$

Summary Lecture 18: Complex Analysis

1. Complex line integral:

$$\int_C [af(z) + bg(z)]dz = a\int_C f(z)dz + b\int_C g(z)dz$$

$$\int_{-C} f(z)dz = -\int_C f(z)dz$$

$$\int_{C_1 \cup C_2} f(z)dz = \int_{C_1} f(z)dz + \int_{C_2} f(z)dz \quad \text{when} \quad C_1 \cap C_2 = \emptyset$$

$$\boxed{\left|\int_C f(z)dz\right| \le M \cdot L} \quad M = \max_{z \in C} |f(z)|, \quad L = \text{length of } C$$

2. Cauchy's integral theorem

f analytic in simply connected domain D, $C \subset D$ simple, closed curve

$$\implies \oint_C f(z)dz = 0$$

3. Corollaries:

(a) $\int_C f(z)dz$ is independent of path in D

(b) The indefinite integral of f exists in D

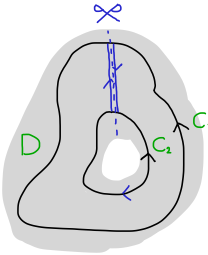
Summary Lecture 18: Complex Analysis

4. Domains with holes:

   Cut to have a simply connected domain...

   ... add segments along cut to have closed curve...

   Then use Cauchy:



Cauchy in the cut domain D^* and cancelations along cut:

$$\oint_{C_1} f(z)dz = \oint_{C_2} f(z)dz$$