Summary: Laplace Transform

1. $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$

2. $L[f](s)$ exists for $s > k$ if

   (A1) f is piece-wise continuous

   (A2) $|f(t)| \leq Me^{kt}$ for some M and k

3. Uniqueness:

   $F(s) = G(s),\ s > k \quad \Leftrightarrow \quad f(t) = g(t),\ \ t \geq 0$ (except discont. points)

$$f : [0,\infty) \to \mathbb{R} \qquad \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\rightleftarrows}} \qquad F : (k,\infty) \to \mathbb{R}$$

Summary: Laplace Transform

4. Examples:

$$\mathcal{L}[\sin \omega t](s) = \frac{\omega}{s^2 + \omega^2}, \; s > 0, \qquad |\sin \omega t| \leq 1 e^{0t}$$

$$\mathcal{L}[e^{at}](s) = \frac{1}{s-a}, \; s > a, \qquad |e^{at}| \leq 1 e^{at}$$

5. Properties:

Linearity: $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at} f(t)](s) = \mathcal{L}[f](s-a)$ for $s - a > k$

Derivatives: $\mathcal{L}[f'(t)](s) = s\mathcal{L}[f](s) - f(0)$

$$\mathcal{L}[f^{(n)}(t)](s) = s^n \mathcal{L}[f](s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$$

Popular and powerful tool to solve linear differential and integral equations

Lecture 2: Laplace Transform

Kreyszig: Sections 6.2 and 6.3

1. Laplace transform of an integral
2. Solving differential equations with the Laplace transform
3. Unit step functions
4. t-shifting
5. Many examples

Homework: Repeat partial fractions and ordinary differential equations.

Godkjenn utdanningsplanen *din i studenweb før torsdag 25.08.!*

Summary: Laplace Transform

1. $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$

2. $f : [0, \infty) \to \mathbb{R} \quad \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\rightleftarrows}} \quad F : (k, \infty) \to \mathbb{R}$

3. Unit step function: ($a \in \mathbb{R}$ fixed)

$$u(t-a) = \begin{cases} 0, & t < a \\ 1, & t > a \end{cases}$$

$$\mathcal{L}[u(t-a)](s) = \frac{1}{s}e^{-as}$$

Summary: Laplace Transform

4. Properties:

Linearity: $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s-a)$ for $s - a > k$

t-shift: $\boxed{\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f](s)}$

Derivatives: $\mathcal{L}[f'(t)](s) = s\mathcal{L}[f](s) - f(0)$

$\mathcal{L}[f''(t)](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$

Integral: $\boxed{\mathcal{L}[\int_0^t f(\tau)d\tau](s) = \frac{1}{s}\mathcal{L}[f](s)}$

5. Solving equations with the Laplace transform: