

Laplace Transform

① $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st} dt$

② Properties:

Linearity: $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s - a)$

t-shift: $\mathcal{L}^{-1}[e^{-as}F(s)](t) = f(t - a)u(t - a)$

Derivatives: $\mathcal{L}[f'(t)](s) = s\mathcal{L}[f](s) - f(0)$

Integral: $\mathcal{L}[\int_0^t f(\tau)d\tau](s) = \frac{1}{s}\mathcal{L}[f](s)$

Convolution: $\mathcal{L}[f * g](s) = \mathcal{L}[f](s)\mathcal{L}[g](s)$ $(f * g)(t) = \int_0^t f(\tau)g(t - \tau)d\tau$

③ Unit step function, delta function:

δ -function: $\int_0^\infty \delta(t - a)f(t)dt = f(a)$ for all f cont. at $t = a (> 0)$

$$\mathcal{L}[u(t - a)] = \frac{1}{s}e^{-as}, \quad \mathcal{L}[\delta(t - a)] = e^{-as}$$

Lecture 4: Laplace Transform

Kreyszig: Sections 6.6, 6.7, 11.1

- 1 Differentiation of transforms
- 2 Integration of transforms
- 3 Systems of differential equations
- 4 Fourier series (introduction)
- 5 Examples

Lecture 4: Laplace Transform

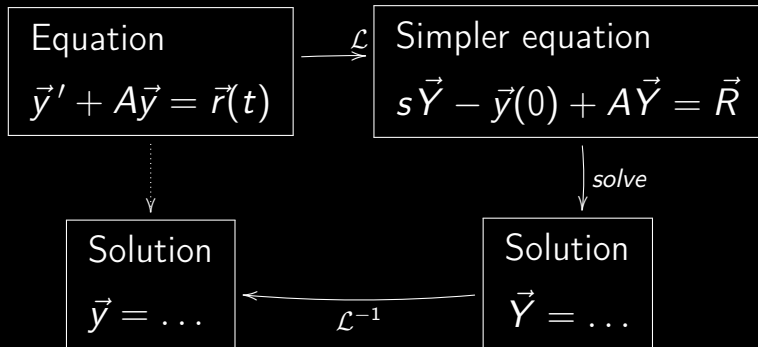
Derivatives and integrals of transforms:

$$-F'(s) = \mathcal{L}[tf(t)](s)$$

$$\int_s^\infty F(\bar{s})d\bar{s} = \mathcal{L}\left[\frac{1}{t}f(t)\right](s)$$

Lecture 4: Laplace Transform

Systems of ordinary differential equations:



Lecture 4: Fourier Analysis

$f(x)$ is p -periodic if $f(x + p) = f(x)$ for all $x \in \mathbb{R}$.

Representation of $f(x)$ by trigonometric series:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right).$$

Summary: Laplace Transform

① $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st} dt$

② **Properties:**

Linearity: $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s - a)$

t-shift: $\mathcal{L}^{-1}[e^{-as}F(s)](t) = f(t - a)u(t - a)$

Derivatives: $\mathcal{L}[f'(t)](s) = s\mathcal{L}[f](s) - f(0)$

$$\mathcal{L}^{-1}[F'(s)](t) = -tf(t)$$

Integral: $\mathcal{L}[\int_0^t f(\tau)d\tau](s) = \frac{1}{s}\mathcal{L}[f](s)$

$$\mathcal{L}^{-1}[\int_s^\infty F(\bar{s})d\bar{s}](t) = \frac{1}{t}f(t)$$

Convolution: $\mathcal{L}[f * g](s) = \mathcal{L}[f](s)\mathcal{L}[g](s) \quad (f * g)(t) = \int_0^t f(\tau)g(t - \tau)d\tau$

③ **Unit step and delta function:** $\mathcal{L}[u(t - a)] = \frac{1}{s}e^{-as}, \quad \mathcal{L}[\delta(t - a)] = e^{-as}$