# Lecture 1: Laplace Transform

Kreyszig: Sections 6.1 and 6.2

1. Defintion of the Laplace transform
2. Existence and uniqueness
3. Properties: Linearity, $s$-shift, derivatives, integral
4. Many examples

**Homework:** Repeat partial fractions and ordinary differential equations.

**Godkjenn utdanningsplanen** *din i studenweb før søndag 25.08.!*

Laplace Transform:

$$F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$$

If $f$ piece-wise continuous and $|f(t)| \leq Me^{kt}$, then

a) $F(s) = \mathcal{L}[f](s)$ exists for $s > k$,

b) and is unique: $F = G \iff f = g$

$$F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$$

Properties:

$$\mathcal{L}[a\,f + b\,g] = a\,\mathcal{L}[f] + b\,\mathcal{L}[g]$$

$$\mathcal{L}[e^{at} f\,](s) = \mathcal{L}[f](s-a)$$

$$\mathcal{L}[f'\,](s) = s\,\mathcal{L}[f](s) - f(0)$$

$$\mathcal{L}[\,\int_0^t f(\tau)\,d\tau\,](s) = \frac{1}{s}\mathcal{L}[f](s)$$

# Summary Lecture 1: Laplace Transform

1. $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$

2. $L[f](s)$ exists for $s > k$ if

   (A1) $f$ is piece-wise continuous

   (A2) $|f(t)| \leq Me^{kt}$ for some $M$ and $k$

3. Uniqueness:

   $F(s) = G(s),\ s > k \quad \Leftrightarrow \quad f(t) = g(t),\ t \geq 0$ (except finite no. of pt's)

$$f : [0, \infty) \to \mathbb{R} \quad \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\rightleftarrows}} \quad F : (k, \infty) \to \mathbb{R}$$

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4. Examples:

$$\mathcal{L}[\sin \omega t](s) = \frac{\omega}{s^2+\omega^2}, \; s > 0, \qquad |\sin \omega t| \leq 1 e^{0t}$$

$$\mathcal{L}[e^{at}](s) = \frac{1}{s-a}, \; s > a, \qquad |e^{at}| \leq 1 e^{at}$$

5. Properties:

Linearity: $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s-a)$ for $s - a > k$

Derivatives: $\mathcal{L}[f'(t)](s) = s\mathcal{L}[f](s) - f(0)$

$$\mathcal{L}[f^{(n)}(t)](s) = s^n\mathcal{L}[f](s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$$

Integral: $\mathcal{L}[\int_0^t f(\tau)d\tau](s) = \frac{1}{s}\mathcal{L}[f](s)$

Popular and powerful tool to solve linear differential and integral equations