

# Lecture 1: Laplace Transform

Kreyszig: Sections 6.1 and 6.2

- ① Definition of the Laplace transform
- ② Existence and uniqueness
- ③ Properties: Linearity,  $s$ -shift, derivatives, integral
- ④ Many examples

**Homework:** Repeat partial fractions and ordinary differential equations.

**Godkjenn utdanningsplanen din i studenweb før søndag 25.08.!**

# Lecture 1: Laplace Transform

Laplace Transform:

$$F(s) = \mathcal{L}[f](s) = \int_0^{\infty} f(t)e^{-st} dt$$

If  $f$  piece-wise continuous and  $|f(t)| \leq M e^{kt}$ , then

- a)  $F(s) = \mathcal{L}[f](s)$  exists for  $s > k$ ,
- b) and is unique:  $F = G \iff f = g$

# Lecture 1: Laplace Transform

$$\mathcal{F}(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$$

Properties:

$$\mathcal{L}[af + bg] = a\mathcal{L}[f] + b\mathcal{L}[g]$$

$$\mathcal{L}[e^{at}f](s) = \mathcal{L}[f](s - a)$$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$

$$\mathcal{L}\left[\int_0^t f(\tau) d\tau\right](s) = \frac{1}{s} \mathcal{L}[f](s)$$

# Summary Lecture 1: Laplace Transform

①  $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st} dt$

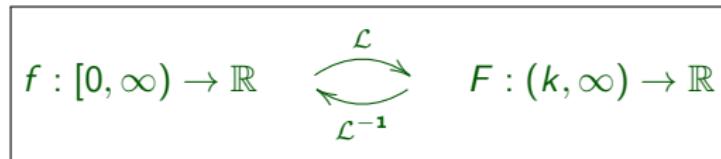
②  $\mathcal{L}[f](s)$  exists for  $s > k$  if

(A1)  $f$  is piece-wise continuous

(A2)  $|f(t)| \leq M e^{kt}$  for some  $M$  and  $k$

③ Uniqueness:

$$F(s) = G(s), s > k \Leftrightarrow f(t) = g(t), t \geq 0 \text{ (except finite no. of pt's)}$$



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## ④ Examples:

$$\mathcal{L}[\sin \omega t](s) = \frac{\omega}{s^2 + \omega^2}, \quad s > 0, \quad |\sin \omega t| \leq 1e^{0t}$$

$$\mathcal{L}[e^{at}](s) = \frac{1}{s - a}, \quad s > a, \quad |e^{at}| \leq 1e^{at}$$

## ⑤ Properties:

Linearity:  $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift:  $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s - a)$  for  $s - a > k$

Derivatives:  $\mathcal{L}[f'(t)](s) = s\mathcal{L}[f](s) - f(0)$

$$\mathcal{L}[f^{(n)}(t)](s) = s^n \mathcal{L}[f](s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$$

Integral:  $\mathcal{L}[\int_0^t f(\tau)d\tau](s) = \frac{1}{s}\mathcal{L}[f](s)$

Popular and powerful tool to solve linear differential and integral equations