

Summary: Complex Analysis

1 Analytic functions and Cauchy-Riemann equations:

$$f(z) = u(x, y) + iv(x, y) \text{ analytic in domain } D$$



u_x, u_y, v_x, v_y exists, are continuous, and

$$\boxed{u_x = v_y \quad \text{and} \quad u_y = -v_x} \quad \text{in} \quad D.$$

2 Laplace equation $u_{xx} + u_{yy} = 0$

$$f(z) = u(x, y) + iv(x, y) \text{ analytic in domain } D$$



u, v are 2 times continuously differentiable, and

$$\boxed{u_{xx} + u_{yy} = 0 \quad \text{and} \quad v_{xx} + v_{yy} = 0} \quad \text{in} \quad D.$$

u, v are **conjugate harmonic** functions.

3 Conformal mappings:

Maps preserving angles and orientation between smooth curves

$$\boxed{f \text{ analytic in } D \Rightarrow f \text{ conformal where } f' \neq 0 \text{ in } D}$$

$$f(z) = z^n \text{ conformal at } z \neq 0, \quad z = 0: \arg(\dot{w}_1 - \dot{w}_2) = n \arg(\dot{z}_1 - \dot{z}_2)$$

Lecture 16: Complex Analysis

Kreyszig: Sections 13.5, 13.6, 13.7

- 1 Exponential function
- 2 Trigonometric and hyperbolic functions
- 3 Logarithm

OBS: Du trenger 8 (av 12) øvinger godkjent for å få ta eksamen!!

Lecture 16: Exponential function

$$\boxed{e^z = e^{x+iy} = e^x(\cos y + i \sin y)} \quad 2\pi i\text{-periodic}$$

$$|e^z| = e^x, \quad |e^{iy}| = 1, \quad \arg e^z = y + 2\pi n, \quad n \in \mathbb{Z}$$

$$(e^z)' = e^z \quad (\text{analytic in } \mathbb{C})$$

$$e^z \neq 0 \text{ in } \mathbb{C} \quad (\text{conformal in } \mathbb{C})$$

Lecture 16: Trigonometric/hyperbolic functions

$$\cos z = \frac{1}{2}(e^{iz} + e^{-iz}), \quad \sin z = \frac{1}{2i}(e^{iz} - e^{-iz}) \quad [p = 2\pi]$$

$$\cosh z = \frac{1}{2}(e^z + e^{-z}), \quad \sinh z = \frac{1}{2}(e^z - e^{-z}) \quad [p = 2\pi i]$$

$$\tan z = \frac{\sin z}{\cos z} \quad \dots$$

$$\cos^2 z + \sin^2 z = 1 \quad \text{and} \quad \cosh^2 z - \sinh^2 z = 1$$

$$(\cos z)' = -\sin z, \dots \quad \text{same as for real functions}$$

Lecture 16: Logarithm

Definition: $w = \ln z \iff e^w = z$

$$\ln z = \ln |z| + i(\operatorname{Arg} z + 2\pi n), \quad n \in \mathbb{Z}$$

$$\operatorname{Ln} z = \ln |z| + i\operatorname{Arg} z \quad (\text{principal value})$$

Summary Lecture 16: Complex Analysis

1 Exponential function:

$$e^z = e^{x+iy} = e^x(\cos y + i \sin y) \quad 2\pi i\text{-periodic}$$

$$|e^z| = e^x, \quad |e^{iy}| = 1, \quad \arg e^z = y + 2\pi n, \quad n \in \mathbb{Z}$$

$$e^{z_1} e^{z_2} = e^{z_1 + z_2}$$

$$(e^z)' = e^z \quad (\text{analytic in } \mathbb{C}), \quad e^z \neq 0 \text{ in } \mathbb{C} \quad (\text{conformal in } \mathbb{C})$$

2 Trigonometric and hyperbolic functions:

$$\cos z = \frac{1}{2}(e^{iz} + e^{-iz}) \quad \sin z = \frac{1}{2i}(e^{iz} - e^{-iz}) \quad 2\pi \text{ periodic}$$

$$\cosh z = \frac{1}{2}(e^z + e^{-z}) \quad \sinh z = \frac{1}{2}(e^z - e^{-z}) \quad 2\pi i \text{ periodic}$$

$$\tan z = \frac{\sin z}{\cos z} \quad \dots$$

$$\cos^2 z + \sin^2 z = 1 \quad \text{and} \quad \cosh^2 z - \sinh^2 z = 1$$

$$(\cos z)' = -\sin z, \dots \quad \text{derivation as for real functions}$$

3 Logarithm:

$$\ln z = \ln |z| + i(\operatorname{Arg} z + 2\pi n), \quad n \in \mathbb{Z}$$

$$\operatorname{Ln} z = \ln |z| + i \operatorname{Arg} z \quad (\text{principal value})$$