

Summary: Complex Analysis

1 Exponential function:

$$e^z = e^{x+iy} = e^x(\cos y + i \sin y) \quad 2\pi i\text{-periodic}$$

$$|e^z| = e^x, \quad |e^{iy}| = 1, \quad \arg e^z = y + 2\pi n, \quad n \in \mathbb{Z}$$

$$e^{z_1} e^{z_2} = e^{z_1 + z_2}$$

$$(e^z)' = e^z \quad (\text{analytic in } \mathbb{C}), \quad e^z \neq 0 \text{ in } \mathbb{C} \quad (\text{conformal in } \mathbb{C})$$

2 Trigonometric and hyperbolic functions:

$$\cos z = \frac{1}{2}(e^{iz} + e^{-iz}) \quad \sin z = \frac{1}{2i}(e^{iz} - e^{-iz}) \quad 2\pi \text{ periodic}$$

$$\cosh z = \frac{1}{2}(e^z + e^{-z}) \quad \sinh z = \frac{1}{2}(e^z - e^{-z}) \quad 2\pi i \text{ periodic}$$

$$\tan z = \frac{\sin z}{\cos z} \quad \dots$$

$$\cos^2 z + \sin^2 z = 1 \quad \text{and} \quad \cosh^2 z - \sinh^2 z = 1$$

$$(\cos z)' = -\sin z, \dots \quad \text{differentiation rules as for real functions}$$

3 Logarithm:

$$\ln z = \ln |z| + i(\operatorname{Arg} z + 2\pi n), \quad n \in \mathbb{Z}$$

$$\operatorname{Ln} z = \ln |z| + i \operatorname{Arg} z \quad (\text{principal value})$$

Lecture 17: Complex Analysis

Kreyszig: Sections 13.7, 14.1

- 1 Logarithm
- 2 Complex Line integral
- 3 Examples

OBS: Du trenger 8 (av 12) øvinger godkjent for å få ta eksamen!!

Lecture 17: Logarithm and general powers

$$\ln z = \ln |z| + i(\operatorname{Arg} z + 2\pi n), \quad n \in \mathbb{Z}$$

$$\operatorname{Ln} z = \ln |z| + i\operatorname{Arg} z \quad (\text{principal value})$$

$\operatorname{Ln} z$ analytic except at $z = 0$ and negativ real axis

$$(\operatorname{Ln} z)' = \frac{1}{z}$$

$$z^c \stackrel{\text{DEF}}{=} e^{c \ln z}$$

Lecture 17: Complex line integral

Defined via Riemann-sums:

$$\int_C f(z) dz = \lim_{n \rightarrow \infty, |P_n| \rightarrow 0} \sum_{k=1}^n f(\tilde{z}_k) \Delta z_k$$

Exists and uniquely defined when:

(A1) C piecewise smooth, oriented, finite length

(A2) f is continuous on C

Along parametric curve $C : z(t), t \in [a, b]$:

$$\int_C f(z) dz = \int_a^b f(z(t)) \dot{z}(t) dt$$

Recall: - Parametric curve $C : z(t), t \in [a, b]$, **smooth** if $\dot{z}(t)$ exists, cont., $\neq 0$
- ... then unit tangent $T(t) = \frac{\dot{z}(t)}{|\dot{z}(t)|}$ exists and is continuous

Lecture 17: Complex line integral

$$\int_C [af(z) + bg(z)]dz = a \int_C f(z)dz + b \int_C g(z)dz$$

$$\int_{-C} f(z)dz = - \int_C f(z)dz$$

$$\int_{C_1 \cup C_2} f(z)dz = \int_{C_1} f(z)dz + \int_{C_2} f(z)dz, \quad C_1 \cap C_2 = \emptyset$$

Summary Lecture 17: Complex Analysis

1 Logarithm:

$$\ln z = \ln |z| + i(\operatorname{Arg} z + 2\pi n), \quad n \in \mathbb{Z}$$

$$\operatorname{Ln} z = \ln |z| + i\operatorname{Arg} z \quad (\text{principal value})$$

$$(\operatorname{Ln} z)' = \frac{1}{z}$$

$$\operatorname{Ln} z \text{ analytic except at } z = 0 \text{ and negative real axis}$$

$$z^c \stackrel{\text{DEF}}{=} e^{c \ln z}$$

2 Complex line integral:

Defined via Riemann-sums, exists and uniquely defined when:

(A1) C piecewise smooth, oriented curve with finite length

(A2) f is continuous on C

$$\int_C f(z) dz = \int_a^b f(z(t)) \dot{z}(t) dt \quad \text{when } C : z(t), t \in [a, b], \text{ smooth}$$

$$\int_C [af(z) + bg(z)] dz = a \int_C f(z) dz + b \int_C g(z) dz$$

$$\int_{-C} f(z) dz = - \int_C f(z) dz$$

$$\int_{C_1 \cup C_2} f(z) dz = \int_{C_1} f(z) dz + \int_{C_2} f(z) dz \quad \text{when } C_1 \cap C_2 = \emptyset$$