Laplace Transform

1. $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$

2. $L[f](s)$ exists for $s > k$ if

   (A1) f is piece-wise continuous

   (A2) $|f(t)| \leq Me^{kt}$ for some M and k

3. Uniqueness:

   $F(s) = G(s),\ s > k \quad \Leftrightarrow \quad f(t) = g(t),\ \ t \geq 0$ (except at some pt's)

$$f : [0, \infty) \to \mathbb{R} \quad \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\rightleftarrows}} \quad F : (k, \infty) \to \mathbb{R}$$

Laplace Transform

4. Examples:

$$\mathcal{L}[\sin \omega t](s) = \frac{\omega}{s^2 + \omega^2}, \; s > 0, \qquad |\sin \omega t| \leq 1e^{0t}$$

$$\mathcal{L}[e^{at}](s) = \frac{1}{s-a}, \; s > a, \qquad |e^{at}| \leq 1e^{at}$$

5. Properties:

Linearity: $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s-a)$ for $s - a > k$

Derivatives: $\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$

$$\mathcal{L}[f^{(n)}](s) = s^n\mathcal{L}[f](s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$$

Integral: $\mathcal{L}[\int_0^t f(\tau)d\tau](s) = \frac{1}{s}\mathcal{L}[f](s)$

Popular and powerful tool to solve linear differential and integral equations

Lecture 2: Laplace Transform

Kreyszig: Sections 6.2 – 6.4

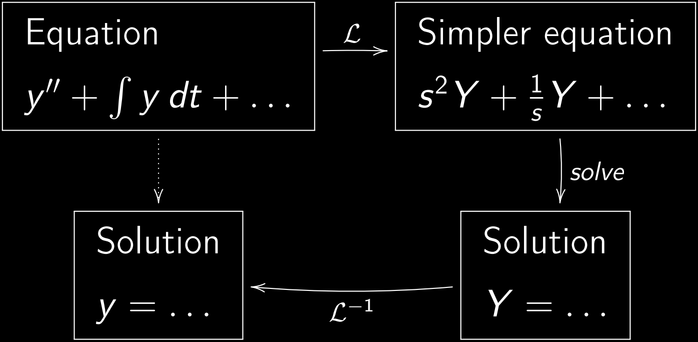
1. Solving differential equations with the Laplace transform
2. Unit step functions
3. t-shifting
4. Delta-functions
5. Examples

Homework: Repeat partial fractions.

Registrer deg som student i faget *i studenweb før søndag 25.08.!*

$$F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$$

Solving equations with the Laplace transform:



$$F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$$

Unit step function

$$u(t-a) = \begin{cases} 0, & t < a \\ 1, & t > a \end{cases}$$

$$\mathcal{L}[u(t-a)](s) = \frac{1}{s}e^{-as}$$

$$F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$$

t-shifting

$$\mathcal{L}[\,f(t-a)u(t-a)\,](s) = e^{-as}\mathcal{L}[f](s)$$

$$\mathcal{L}^{-1}[\,e^{-as}F(s)\,](t) = f(t-a)u(t-a)$$

$$F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$$

Delta-function: (unit impuls)

$$\int_0^\infty \delta(t-a)f(t)\,dt = f(a) \quad \text{for all} \quad f \text{ cont.}$$

$$\mathcal{L}[\delta(t-a)] = e^{-as}$$

Summary Lecture 2: Laplace Transform

1. $F(s) = \mathcal{L}[f](s) = \int_0^\infty f(t)e^{-st}dt$

2. $f : [0, \infty) \to \mathbb{R} \quad \underset{\mathcal{L}^{-1}}{\overset{\mathcal{L}}{\rightleftarrows}} \quad F : (k, \infty) \to \mathbb{R}$

3. Unit step function: ($a \in \mathbb{R}$ fixed)

$$u(t-a) = \begin{cases} 0, & t < a \\ 1, & t > a \end{cases}$$

4. Delta-function: (unit impuls)

$$\int_0^\infty \delta(t-a)f(t)\,dt = f(a) \quad \text{for all} \quad f \text{ cont. at } t = a\,(> 0)$$

5. $\boxed{\mathcal{L}[u(t-a)] = \frac{1}{s}e^{-as}}$, $\boxed{\mathcal{L}[\delta(t-a)] = e^{-as}}$

Summary Lecture 2: Laplace Transform

4. Properties:

Linearity: $\mathcal{L}[af(t) + bg(t)](s) = a\mathcal{L}[f](s) + b\mathcal{L}[g](s)$

s-shift: $\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f](s-a)$ for $s - a > k$

t-shift: $\boxed{\mathcal{L}[\,f(t-a)u(t-a)\,](s) = e^{-as}\mathcal{L}[f](s)}$

Derivatives: $\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$

$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$

Integral: $\mathcal{L}[\,\int_0^t f(\tau)d\tau\,](s) = \frac{1}{s}\mathcal{L}[f](s)$

5. Solving equations with the Laplace transform: