Laplace transform

$f(t)$: $\quad F(s) = \mathcal{L}f = \int_0^\infty f(t)\, e^{-ts}\, dt$ – direct

$\quad f = \mathcal{L}^{-1} F$ – inverse

Properties:

$\mathcal{L}(af + bg) = a\mathcal{L}f + b\mathcal{L}g$ – linearity

$\mathcal{L}(e^{at} f(t)) = F(s-a)$ – 1st shift theorem

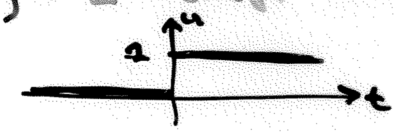
$\mathcal{L}(f') = -f(0) + sF(s)$ ← L-transform of derivative

$\mathcal{L}(f^{(n)}) = -f^{(n-1)}(0) - s f^{(n-2)}(0) - \dots - s^{n-1} f(0) + s^n F(s)$

$\mathcal{L}\left(\int_0^t f(\tau)\, d\tau\right) = \frac{1}{s} F(s)$ ← L.transform of integral.

$\mathcal{L}(f(t-a)\, u(t-a)) = e^{-as} F(s)$ – 2nd shift theorem.

↖ Heaviside f-n :



$\mathcal{L}(t f(t)) = -F'(s)$

$\mathcal{L}\left(\frac{1}{t} f(t)\right) = \int_s^\infty F(\sigma)\, d\sigma$

$f(t), g(t) = 0,\ t < 0$: $\quad f * g(t) = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$

$\mathcal{L}(f * g) = F(s) \cdot G(s)$