# Formlar i numerikk

- La $p(x)$ vere polynomet av grad $\leq n$ som interpolerer $f(x)$ i punkta $x_i$, $i = 0, 1, \ldots, n$. Dersom $x$ og alle nodane ligg i intervallet $[a, b]$, har vi at

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) \prod_{i=0}^{n} (x - x_i)$$

Dersom nodane er jamnt fordelte (inkludert endepunktane), og $|f^{(n+1)}(x)| \leq M$, har vi at

$$|f(x) - p(x)| \leq \frac{1}{4(n+1)} M \left( \frac{b-a}{n} \right)^{n+1}$$

- Numerisk derivasjon:

$$\begin{aligned}
f'(x) &= \frac{1}{h}(f(x+h) - f(x)) - \frac{1}{2} h f''(\xi) \\
f'(x) &= \frac{1}{h}(f(x) - f(x-h)) + \frac{1}{2} h f''(\xi) \\
f'(x) &= \frac{1}{2h}(f(x+h) - f(x-h)) + \frac{1}{2} h^2 f'''(\xi) \\
f''(x) &= \frac{1}{h^2}(f(x+h) - 2f(x) + f(x-h)) - \frac{h^2}{12} f^{(4)}(\xi)
\end{aligned}$$

- Newtons metode for likningssystemet $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ er gitt ved

$$\begin{aligned}
\mathbf{J}^{(k)} \cdot \Delta \mathbf{x}^{(k)} &= -\mathbf{f}(\mathbf{x}^{(k)}) \\
\mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}
\end{aligned}$$

- Iterative teknikkar for løysing av eit lineært likningssystem

$$\sum_{j=1}^{n} a_{ij} x_j = b_i, \qquad i = 1, 2, \ldots, n$$

$$\begin{aligned}
\text{Jacobi :} \quad x_i^{(k+1)} &= \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right) \\
\text{Gauss-Seidel :} \quad x_i^{(k+1)} &= \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right)
\end{aligned}$$

- Heuns metode for løysing av $\mathbf{y}' = \mathbf{f}(x, \mathbf{y})$:

$$\begin{aligned}
\mathbf{K}_1 &= h \mathbf{f}(x_n, \mathbf{y}_n) \\
\mathbf{K}_2 &= h \mathbf{f}(x_n + h, \mathbf{y}_n + \mathbf{K}_1) \\
\mathbf{y}_{n+1} &= \mathbf{y}_n + \frac{1}{2} (\mathbf{K}_1 + \mathbf{K}_2)
\end{aligned}$$

Sjå og formlane i Rottmann.