

## Numerical formulas

- Let  $p(x)$  be the polynomial of degree  $\leq n$  which coincides with  $f(x)$  at points  $x_i, i = 0, 1, \dots, n$ . Under the assumption that  $x$  and all the nodes  $x_i$  lie in the interval  $[a, b]$ , we have

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) \prod_{i=0}^n (x - x_i)$$

If the nodes are evenly spaced, including end points, and  $|f^{(n+1)}(x)| \leq M$ , we have

$$|f(x) - p(x)| \leq \frac{1}{4(n+1)} M \left( \frac{b-a}{n} \right)^{n+1}$$

- Numerical differentiation:

$$f'(x) = \frac{1}{h} (f(x+h) - f(x)) - \frac{h}{2} f''(\xi)$$

$$f'(x) = \frac{1}{h} (f(x) - f(x-h)) + \frac{h}{2} f''(\xi)$$

$$f'(x) = \frac{1}{2h} (f(x+h) - f(x-h)) - \frac{h^2}{6} f'''(\xi)$$

$$f''(x) = \frac{1}{h^2} (f(x+h) - 2f(x) + f(x-h)) - \frac{h^2}{12} f^{(4)}(\xi)$$

- Newton's method for solving the system of equations  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$  is given by the scheme

$$\begin{aligned} \mathbf{J}^{(k)} \cdot \Delta \mathbf{x}^{(k)} &= -\mathbf{f}(\mathbf{x}^{(k)}) \\ \mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)} \end{aligned}$$

- Iteration methods for solving systems of linear equations

$$\sum_{j=1}^n a_{ij} x_j = b_i, \quad i = 1, 2, \dots, n$$

$$\text{Jacobi: } x_i^{(k+1)} = \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)} \right)$$

$$\text{Gauss-Seidel: } x_i^{(k+1)} = \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)} \right)$$

- Heun's method for solving  $\mathbf{y}' = \mathbf{f}(x, \mathbf{y})$ :

$$\mathbf{K}_1 = h \mathbf{f}(x_n, \mathbf{y}_n)$$

$$\mathbf{K}_2 = h \mathbf{f}(x_n + h, \mathbf{y}_n + \mathbf{K}_1)$$

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{2} (\mathbf{K}_1 + \mathbf{K}_2)$$

See formulas in Rottmann as well.

### Table of Laplace transforms

$f(t)$	$\mathcal{L}(f)$
1	$\frac{1}{s}$
$t$	$\frac{1}{s^2}$
$t^n$ ( $n = 0, 1, 2, \dots$ )	$\frac{n!}{s^{n+1}}$
$e^{at}$	$\frac{1}{s-a}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
$\sinh at$	$\frac{a}{s^2 - a^2}$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$