TMA4123 / TMA4125
Matematikk 4M/N

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Exercise set 2

Kreyzsig (8th ed): 10.3.9 Find the Fourier series of the periodic function $f(x)$, of period $p = 2L$, and sketch $f(x)$ and the first three partial sums. (Show the details of your work.)

$$f(x) = \begin{cases} 0 & \text{if} \quad -1 < x < 0 \\ x & \text{if} \quad 0 < x < 1 \end{cases}, \quad p = 2L = 2$$

Kreyzsig (8th ed): 10.4.2 Are the following functions odd, even, or neither odd or even?

$$x + x^2, \ |x|, \ e^x, \ e^{x^2}, \ \sin^2 x, \ x \sin x, \ \ln x, \ x \cos x, \ e^{-|x|}$$

Kreyzsig (8th ed): 10.4.7 Are the following function $f(x)$, which is assumed to be periodic, of period 2π, even, odd or neither even or odd?

$$f(x) = \begin{cases} 0 & \text{if} \quad 2 < x < 2\pi - 2 \\ x & \text{if} \quad -2 < x < 2 \end{cases}$$

Kreyzsig (8th ed): 10.4.13 State whether the given function is even or odd. Find its Fourier series. Sketch the function and some partial sums. (Show the details of your work.)

$$f(x) = \begin{cases} x & \text{if} \quad -\pi/2 < x < \pi/2 \\ \pi - x & \text{if} \quad \pi/2 < x < 3\pi/2 \end{cases}$$

Kreyzsig (8th ed): 10.4.15 State whether the given function is even or odd. Find its Fourier series. Sketch the function and some partial sums. (Show the details of your work.)

$$f(x) = x^2/2, \quad -\pi < x < \pi$$

Kreyzsig (8th ed): 10.4.19 Use problem 15 to show that

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + - \ldots = \frac{\pi^2}{12}.$$

Kreyzsig (8th ed): 10.4.23 | Find the Fourier cosine series as well as the Fourier sine series. Sketch $f(x)$ and its two periodic extensions. (Show the details.)

$$f(x) = \pi - x, \quad 0 < x < \pi$$

8 | Let f be a 2π-periodic function and

$$f(x) \sim a_0 + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$$

be its Fourier series. Find the Fourier series of $f(-x)$.

9 | Verify the formulas for the coefficients in the half-range expansions. (In the same way as we did the formula for a_0 in class.)

Kreyzsig (8th ed): 12.1.4 | Let $z_1 = 4 + 3i$ and $z_2 = 2 - 5i$. Find the following in the form $x + iy$, showing the details of your work:

$$(3z_1 - z_2)^2$$

Kreyzsig (8th ed): 12.1.7 | Let $z_1 = 4 + 3i$ and $z_2 = 2 - 5i$. Find the following in the form $x + iy$, showing the details of your work:

$$\text{Re}\left(z_1^3\right), (\text{Re}\, z_1)^3$$

Kreyzsig (8th ed): 12.1.9 | Let $z_1 = 4 + 3i$ and $z_2 = 2 - 5i$. Find the following in the form $x + iy$, showing the details of your work:

$$z_1 \bar{z}_2, \, \bar{z}_1 z_2$$

Kreyzsig (8th ed): 12.1.13 | Let $z = x + iy$. Find (showing the details of your work)

$$\text{Im}\,(1/z)$$

Kreyzsig (8th ed): 12.1.15 | Let $z = x + iy$. Find (showing the details of your work)

$$(1 + i)^{16}$$