Norges teknisk-naturvitenskapelige universitet

Institutt for matematiske fag

Contact during the exam:
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EXAM IN CALCULUS 4N (TMA4125)
Monday June 06, 2005
Time: 09:00 –13:00

Hjelpemidler: Simple calculator (HP 30S), Rottmann: matematisk formelsamling

Grades: 27.06.05

Oppgave 1

a) Find the inverse Laplace transform of the function

$$F(s) = \frac{s+4}{(s+2)^2}$$

b) Solve the initial value problem:

$y''(t) + 4y'(t) + 4y(t) = 0, \ t \geq 0, \ y(0) = 1, \ y'(0) = 0.$

c) Solve the integral equation:

$y(t) + \int_0^t e^{-2(t-\tau)} y(\tau)d\tau = e^{-2t}, \ t > 0.$

Oppgave 2

a) Find Fourier series for 2π-periodic even function $f(t)$ such that $f(x) = \frac{\pi}{2} - x, \ 0 < x < \pi$.

b) Find all solutions of the form $u(x,t) = X(x)T(t)$ for the problem:

(1) $\frac{\partial^2 u}{\partial x^2} - 2u - \frac{\partial u}{\partial t} = 0, \ 0 < x < \pi, \ t > 0.$

(2) $u_x(0,t) = 0, \ u_x(\pi,t) = 0, \ t > 0.$

c) Find the solution to the problem $(1), (2)$ in part **b.**) which also satisfies the initial condition

$u(x,0) = \frac{\pi}{2} - x, \ 0 < x < \pi.$

Oppgave 3

Find complex Fourier transform of the function $f(x) = e^{-|x|}$ and then find the value of the integral

$$\int_{-\infty}^{\infty} \frac{\cos \omega}{1 + \omega^2} d\omega$$

Oppgave 4

a) Find a polynomial of the smallest possible degree which solves the interpolation problem

x_k	-2	-1	0	1	2
$p(x_k)$	6	0	0	0	15

b) Let $p(x)$ be the polynom from part **a**). Using the Simpsom method with step 1 evaluate the integral $\int_{-2}^{2} p(x)dx$.

Oppgave 5

We are solving partial differential equation

$$u_t = u_{xx}, \quad -1 \leq \quad x \leq 1, \quad t \geq 0$$

$$u(x,0) = 1 - x^2,$$

$$u(-1,t) = 0, \quad u(1,t) = 0, \quad t \geq 0.$$

Let $k = 0.5, h = 0.5$. Using the Crank-Nikolson method write down the system of linear equations for the values

$u_{11} \approx u(-0.5, 0.5)$, $u_{21} \approx u(0, 0.5)$, $u_{31} \approx u(0.5, 0.5)$.