Løsningsforslag
Eksamen 06.08.99 i SIF5013/14

Oppgave 1

$$c_{n+2} = \frac{-n^2+5n-6}{(n+2)(n+1)} = \frac{-(n-2)(n-3)}{(n+2)(n+1)}$$

$$c_0 = c_1 = 1,\ c_2 = -3,\ c_3 = -1/3$$

$$\underline{\underline{y = 1 + x - 3x^2 - \tfrac{1}{3}x^3}}$$

Oppgave 2

a)

$$\mathcal{L}^{-1}\left(\frac{as+b}{(x+2)^2+4}\right) = \mathcal{L}^{-1}\left(\frac{a(s+2)+b-2a}{(s+2)^2+4}\right) = \underline{\underline{e^{-2t}(a\cos 2t + \left(\frac{b}{2}-a\right)\sin 2t)}}$$

b) Laplacetransformerer:

$$(s^2+4s+8)Y - 2s - 9 = \frac{s^2+4s+8}{s^3}e^{-2s}$$

$$Y(s) = \frac{2s+9}{(s+2^2+4} + \frac{1}{s^3}e^{-2s}$$

$$\underline{\underline{y(t) = e^{-2t}(2\cos 2t + \tfrac{5}{2}\sin 2t) + \tfrac{1}{2}(t-2)^2 u(t-2)}}$$

Oppgave 3

a) Setter inn:

$$h'(t)\sin ax = -\kappa h(t) a^2 \sin ax \Rightarrow h(t) = Ae^{-\kappa a^2 t}$$

$$h(0) = 1 \Rightarrow \underline{\underline{h(t) = e^{-\kappa a^2 t}}}$$

b) Fourierrekken (sinusrekke p.g.a. odde funksjon)

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\int_0^{\pi/2} 1\cdot\sin nx\,dx = \frac{2}{\pi n}\left(1-\cos\frac{\pi n}{2}\right)$$

Fra resultatet i **a)** følger

$$\underline{\underline{u(x,t) = \sum_{n=1}^{\infty}\frac{2}{\pi n}\left(1-\cos\frac{\pi n}{2}\right)e^{-\kappa n^2 t}\sin nx}}$$

Oppgave 4

a)

$$T(af_1+bf_2) = af_1(t) + bf_2(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+(t-\tau)^2}(af_1(\tau)+bf_2(\tau))\,d\tau$$
$$= a\Big(f_1(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+(t-\tau)^2}f_1(\tau)\,d\tau\Big)$$
$$+ b\Big(f_2(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+(t-\tau)^2}f_2(\tau)\,d\tau\Big)$$
$$= aTf_1 + bTf_2$$

Fouriertransformerer:

$$\hat{g}(\omega) = \hat{f}(\omega) + \mathcal{F}\left(\frac{1}{\pi}\frac{1}{1+t^2}\right)\cdot\hat{f}(\omega) = \hat{f}(\omega) + e^{-|\omega|}\hat{f}(\omega) = (1+e^{-|\omega|})\hat{f}(\omega)$$

$\underline{\underline{\text{Transferfunksjon: } 1+e^{-|\omega|}}}$

(OBS: Feil i oppgaveteksten, $1+e^{|\omega|}$, ble rettet på eksamen.)

b)

$$\hat{g}(\omega) = (1+e^{-|\omega|})\hat{f}(\omega) \Rightarrow \hat{f}(\omega) = (i\omega)e^{-\omega^2/2}$$

$$\underline{\underline{f(t)}} = \frac{d}{dt}\left(\frac{1}{\sqrt{2\pi}}e^{-t^2/2}\right) = \underline{\underline{-\frac{1}{\sqrt{2\pi}}te^{-t^2/2}}}$$

Oppgave 5

a) Tabell over dividerte differanser:

x	f[ ]	f[,]	f[,,]	
-1	-4			
		3		
0	-1		-1	
		1		1
1	0		2	
		5		
2	5			

Polynomet blir:

$$p(x) = -4 + 3(x+1) - (x+1)x + (x+1)x + (x+1)x(x-1)$$
$$= \underline{\underline{x^3 - x^2 + x - 1}}$$

b) La det nye polynomet være

$$q(x) = p(x) + c\cdot(x+1)(x-1)x(x-2)$$

Dette interpolerer datapunktene. I tillegg er

$$q^{(4)}(x) = 4!c = 24 \Rightarrow c = 1$$

og

$$\underline{\underline{q(x) = x^4 - x^3 - 2x^2 + 3x - 1}}$$

Oppgave 6

a) Sett $y_1 = x$, $y_2 = x'$
dvs. at systemet blir

$$\begin{aligned} y_1' &= y_2 \qquad & y_1(0) = 0 \\ y_2' &= \cos(y_1) \qquad & y_2(0) = 1 \end{aligned}$$

b) Bruk av 3. ordens RK-metode gir:
1. skritt:

$$k_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \ k_2 = \begin{bmatrix} 0.11 \\ 0.09950 \end{bmatrix}, \ y_1 = \begin{bmatrix} 0.1050 \\ 1.0998 \end{bmatrix} \approx y(0.1)$$

2. skritt:

$$k_1 = \begin{bmatrix} 0.10998 \\ 0.09945 \end{bmatrix}, \ k_2 = \begin{bmatrix} 0.11992 \\ 0.09770 \end{bmatrix}, \ y_2 = \begin{bmatrix} 0.2199 \\ 1.1983 \end{bmatrix} \approx y(0.2)$$

dvs:

$$\underline{\underline{x(0.2) \approx 0.2199.}}$$