

Formler i numerikk

- La $p(x)$ være polynomet av grad $\leq n$ som interpolerer $f(x)$ i punktene $x_i, i = 0, 1, \dots, n$. Forutsatt at x og alle nodene ligger i intervallet $[a, b]$, så gjelder

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) \prod_{i=0}^n (x - x_i)$$

- Newtons dividerte differenser interpolasjonsformel:

$$f(x) \approx f_0 + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] \\ + \dots + (x - x_0)(x - x_1) \dots (x - x_{n-1})f[x_0, \dots, x_n]$$

- Numerisk derivasjon:

$$f'(x) = \frac{1}{h}(f(x+h) - f(x)) - \frac{h}{2}f''(\xi) \\ f'(x) = \frac{1}{2h}(f(x+h) - f(x-h)) - \frac{h^2}{6}f'''(\xi) \\ f''(x) = \frac{1}{h^2}(f(x+h) - 2f(x) + f(x-h)) - \frac{h^2}{12}f^{(4)}(\xi)$$

- Simpsons integrasjonsformel

$$\int_{x_0}^{x_2} f(x) dx \approx h \left(\frac{1}{3}f_0 + \frac{4}{3}f_1 + \frac{1}{3}f_2 \right)$$

- Newtons metode for ligningssystemet $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ er gitt ved

$$\mathbf{J}^{(k)} \cdot \Delta \mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)}) \\ \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}$$

- Iterative teknikker for løsning av et lineært ligningssystem

$$\sum_{j=1}^n a_{ij}x_j = b_i, \quad i = 1, 2, \dots, n$$

$$\text{Jacobi : } x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k)} - \sum_{j=i+1}^n a_{ij}x_j^{(k)} \right)$$

$$\text{Gauss-Seidel : } x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij}x_j^{(k)} \right)$$

- Heuns metode for løsning av $\mathbf{y}' = \mathbf{f}(x, \mathbf{y})$:

$$\mathbf{K}_1 = h \mathbf{f}(x_n, \mathbf{y}_n) \\ \mathbf{K}_2 = h \mathbf{f}(x_n + h, \mathbf{y}_n + \mathbf{K}_1) \\ \mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{2}(\mathbf{K}_1 + \mathbf{K}_2)$$

Tabell over noen Laplacetransformer

| $f(t)$ | $\mathcal{L}(f)$ |
|--------------------------------|-------------------------------------|
| 1 | $\frac{1}{s}$ |
| t | $\frac{1}{s^2}$ |
| t^n ($n = 0, 1, 2, \dots$) | $\frac{n!}{s^{n+1}}$ |
| e^{at} | $\frac{1}{s-a}$ |
| $\cos \omega t$ | $\frac{s}{s^2 + \omega^2}$ |
| $\sin \omega t$ | $\frac{\omega}{s^2 + \omega^2}$ |
| $\cosh at$ | $\frac{s}{s^2 - a^2}$ |
| $\sinh at$ | $\frac{a}{s^2 - a^2}$ |
| $e^{at} \cos \omega t$ | $\frac{s-a}{(s-a)^2 + \omega^2}$ |
| $e^{at} \sin \omega t$ | $\frac{\omega}{(s-a)^2 + \omega^2}$ |

Tabell over noen Fourierretransformer

| $f(t)$ | $\mathcal{F}(f)$ |
|-----------------|--|
| $g(x) = f(ax)$ | $\hat{g}(w) = \frac{1}{a} \hat{f}\left(\frac{w}{a}\right)$ |
| $u(x) - u(x-a)$ | $\frac{1}{\sqrt{2\pi}} \left(\frac{\sin aw}{w} - i \frac{1 - \cos aw}{w} \right)$ |
| $u(x)e^{-x}$ | $\frac{1}{\sqrt{2\pi}} \left(\frac{1}{1+w^2} - i \frac{w}{1+w^2} \right)$ |
| e^{-ax^2} | $\frac{1}{\sqrt{2a}} e^{-\frac{w^2}{4a}}$ |