

Numerical formulas

- Let $p(x)$ be the polynomial of degree $\leq n$ which coincides with $f(x)$ at points $x_i, i = 0, 1, \dots, n$. Under the assumption that x and all the nodes x_i lie in the interval $[a, b]$, we have

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) \prod_{i=0}^n (x - x_i)$$

- Newton's divided difference interpolation formula:

$$f(x) \approx f_0 + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] \\ + \dots + (x - x_0)(x - x_1) \dots (x - x_{n-1})f[x_0, \dots, x_n]$$

- Numerical differentiation:

$$f'(x) = \frac{1}{h}(f(x+h) - f(x)) - \frac{h}{2}f''(\xi)$$

$$f'(x) = \frac{1}{2h}(f(x+h) - f(x-h)) - \frac{h^2}{6}f'''(\xi)$$

$$f''(x) = \frac{1}{h^2}(f(x+h) - 2f(x) + f(x-h)) - \frac{h^2}{12}f^{(4)}(\xi)$$

- Simpson's rule of integration

$$\int_{x_0}^{x_2} f(x) dx \approx h \left(\frac{1}{3}f_0 + \frac{4}{3}f_1 + \frac{1}{3}f_2 \right)$$

- Newton's method for solving the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ is given by the scheme

$$\mathbf{J}^{(k)} \cdot \Delta \mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)}) \\ \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}$$

- Iteration methods for solving systems of linear equations

$$\sum_{j=1}^n a_{ij}x_j = b_i, \quad i = 1, 2, \dots, n$$

$$\text{Jacobi: } x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k)} - \sum_{j=i+1}^n a_{ij}x_j^{(k)} \right)$$

$$\text{Gauss-Seidel: } x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij}x_j^{(k)} \right)$$

- Heun's method for solving $\mathbf{y}' = \mathbf{f}(x, \mathbf{y})$:

$$\mathbf{K}_1 = h\mathbf{f}(x_n, \mathbf{y}_n)$$

$$\mathbf{K}_2 = h\mathbf{f}(x_n + h, \mathbf{y}_n + \mathbf{K}_1)$$

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{2}(\mathbf{K}_1 + \mathbf{K}_2)$$

Table of some Laplace transforms

$f(t)$	$\mathcal{L}(f)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n ($n = 0, 1, 2, \dots$)	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
$\sinh at$	$\frac{a}{s^2 - a^2}$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$

Table of some Fourier transforms

$f(t)$	$\mathcal{F}(f)$
$g(x) = f(ax)$	$\widehat{g}(w) = \frac{1}{a} \widehat{f}\left(\frac{w}{a}\right)$
$u(x) - u(x-a)$	$\frac{1}{\sqrt{2\pi}} \left(\frac{\sin aw}{w} - i \frac{1 - \cos aw}{w} \right)$
$u(x)e^{-x}$	$\frac{1}{\sqrt{2\pi}} \left(\frac{1}{1+w^2} - i \frac{w}{1+w^2} \right)$
e^{-ax^2}	$\frac{1}{\sqrt{2a}} e^{-\frac{w^2}{4a}}$