Norwegian University of Science and Technology
Departement of Mathematical Sciences

TMA4130
Calculus 4N
Fall 2013

Exercise set 11

In exercises 1-3, feel free to use MATLAB to do the computations.

1 Formulate Newton's method for the system

$$x^2 + xy^3 - 9 = 0,$$
$$3x^2y - y^3 - 4 = 0.$$

Use start values $x_0 = 1.2$ and $y_0 = 2.5$ and compute the solution after one and two iterations.

2 Consider the initial value problem

$$y' = -y, \qquad x \in [0,1], \qquad y(0) = 1.$$

a) What is the exact solution $y(x)$ of this problem? What is the value $y(1)$?

b) Solve the problem numerically using the first order Euler forward method. Use 4 steps with $h = 0.25$. Compare the numerical solution at $x = 1$ with the exact value.

c) Solve the problem numerically using the second order Heun's method. Use 2 steps with $h = 0.5$. Compare the numerical solution at $x = 1$ with the exact value.

d) Solve the problem numerically using the classical 4th order Runge-Kutta method. Use one step with $h = 1$. Compare the numerical solution at $x = 1$ with the exact value.

3 Consider now the initial value problem

$$y' = \frac{y}{x} - \left(\frac{y}{x}\right)^2, \qquad x \in [1,2], \qquad y(1) = 1.$$

a) Verify that the exact solution to this problem is given as $y(x) = \frac{x}{1+\ln x}$.

b) Solve the problem numerically using the first order Euler forward method. Use 4 steps with $h = 0.25$. Compare the numerical solution at $x = 2$ with the exact value.

c) Solve the problem numerically using the second order Heun's method. Use 2 steps with $h = 0.5$. Compare the numerical solution at $x = 2$ with the exact value.

d) Solve the problem numerically using the classical 4th order Runge-Kutta method. Use one step with $h = 1$. Compare the numerical solution at $x = 2$ with the exact value.

4 **a)** Skriv $y'' - \cos y = 0$ som et system av førsteordens ODE.

b) Sett opp Eulers metode for systemet i **a)** (dere trenger ikke gjøre noen iterasjoner).

c) Skriv systemet

$$
\begin{aligned}
y_1' &= y_2 \\
y_2' &= y_3 \\
y_3' &= \cos y_1 + \sin y_2 - \mathrm{e}^{y_3} + x^2
\end{aligned}
$$

som en tredjeordens ODE.

d) Hvilke type initialbetingelser trenger vi for å kunne løse systemet i **c)** numerisk?

Repetisjon

5 La f være en 2π-periodisk funksjon der

$$
f(x) = \begin{cases} -x, & \text{hvis } -\pi < x \leq 0, \\ x, & \text{hvis } 0 < x \leq \pi. \end{cases}
$$

a) Finn Fourier-rekka til f.

b) Ved å bruke Fourier-koeffisientene fra forrige punkt, vis at

$$
1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \ldots = \frac{\pi^4}{96}.
$$

Hint. Bruk Parsevals identitet.