Norwegian University of Science
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Department of Mathematical
Sciences

TMA4130
Matematikk 4N
Fall 2013

Exercise set 1

In all problems you are supposed to show the details of your work and describe what you are doing.

1 Consider the 2π-periodic function given by $f(x) = 3 + 5\cos 10x - 3\sin 2x$ for $x \in (-\pi, \pi)$. Find the Fourier coefficients for f by using the explicit formulas together with the orthogonality relations.

2 Plot or sketch the 2π-periodic function that satisfies $f(x) = x^3$ for $x \in (-\pi, \pi)$. Without finding the Fourier series for f, determine the value of f:s Fourier series at $x = \pi$.

3 Let f be periodic with period 4 and be given on $(-2, 2)$ by

$$f(x) = \begin{cases} 0 & -2 < x < 0, \\ x & 0 < x < 2. \end{cases}$$

Find the Fourier series for f.

4 Let f be the 2-periodic function given by $f(x) = x^2$ for $x \in (-1, 1)$. Find the Fourier series for f and sketch f and the first three partial sums in the series.

5 Use the previous problem to establish the formula

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots = \frac{\pi^2}{12}.$$

6 Find the 4-periodic cos- and sin-series for f, where $f(x) = 2 - x$ for $x \in (0, 2)$.