

TMA4130 Matematikk 4N Fall 2013

Exercise set 5

In all problems you are supposed to show the details of your work and describe what you are doing.

1 Finn Fouriertransformasjonen til f(x).

$$f(x) = \begin{cases} -1 & \text{for } -1 < x < 0\\ 1 & \text{for } 0 < x < 1\\ 0 & \text{ellers} \end{cases}$$

2 Solve the integral equation

$$f - f * (e^{-3x}H(x)) = e^{-x}H(x),$$

where H(x) is the Heaviside function

$$H(x) = \begin{cases} 0 & \text{when } x < 0, \\ 1 & \text{when } x > 0, \end{cases}$$

and * stands for the convolution.

3 Compute the Fourier transform of the function

$$f(x) = \begin{cases} 1 & \text{for } |x| \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Use this to evaluate

$$\int_0^\infty \frac{\sin w \cos \frac{1}{2}w}{w} \mathrm{d}w.$$

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- a) Find the Fourier transform $\hat{u}(w,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x,t) e^{-ixw} dx$ of the general solution u(x,t) of the partial differential equation

$$u_t = u_{xx} - u,\tag{1}$$

which satisfies the boundary conditions

$$\lim_{|x|\to\infty} u(x,t) = \lim_{|x|\to\infty} u_x(x,t) = \lim_{|x|\to\infty} u_{xx}(x,t) = \lim_{|x|\to\infty} u_t(x,t) = 0 \quad \text{for all } t \ge 0.$$
(2)

b) Determine the solution u(x,t) of equation (1) which satisfies (2), and the initial condition

$$u(x,0) = e^{-\frac{1}{2}x^2}.$$