Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4130
Matematikk 4N
Fall 2015

**Exercise set 5**

In all problems you are supposed to show the details of your work and describe what you are doing.

**1** Compute the Fourier transform of the following functions

1. $f(x) = x^2 e^{-x^2}$ (hint: use the fact that $\mathcal{F}(e^{-x^2}) = \frac{1}{\sqrt{2}} e^{-\frac{w^2}{4}}$).
2. $f(x) = \begin{cases} x & \text{for } -1 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$
3. $f(x) = \begin{cases} \cos(x) & \text{for } -\pi/2 < x < \pi/2, \\ 0 & \text{otherwise.} \end{cases}$

**2** One can show that the Fourier transform of the function

$$f(x) = \frac{x}{1+x^2} \tan x$$

is the function

$$\hat{f}(\omega) = \frac{\sqrt{2\pi} \cosh \omega}{1+e^2}.$$

Use this result in order to compute the integral

$$\int_{-\infty}^{\infty} \frac{x \sin x}{1+x^2}\, dx.$$

**3** Use the Fourier transform to find functions $u(x,t)$, $x \in \mathbb{R}$, $t > 0$, satisfying the following PDEs:

1. The PDE $u_{tt} = u_{xx}$ with initial conditions

$$u(x,0) = \begin{cases} x & \text{for } -1 < x < 1, \\ 0 & \text{otherwise} \end{cases} \qquad \text{and} \qquad u_t(x,0) = 0.$$

2. The PDE $tu_x - u_t = 0$ with initial condition $u(x,0) = \sin(x)$.
3. The PDE $u_{tt} = 2u_{xx}$ with initial conditions $u(x,0) = \frac{1}{1+x^2}$ and $u_t(x,0) = 0$. Recall that $\mathcal{F}(\frac{1}{1+x^2}) = \sqrt{\frac{2}{\pi}} e^{|w|}$.