Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4130 Matematikk 4N
Fall 2015

Exercise set 2

In all problems you are supposed to show the details of your work and describe what you are doing.

1 **a)** Compute the Fourier series of the 2π-periodic function $f\colon \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x$ for $-\pi < x < \pi$.

b) Use the Parseval formula to compute

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

2 **a)** Compute the Fourier coefficients of the 4-periodic function $f\colon \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 3 & \text{for } -2 < x < 0, \\ -1 & \text{for } 0 < x < 2. \end{cases}$$

b) Compute the Fourier coefficients of the 4π-periodic function $g\colon \mathbb{R} \to \mathbb{R}$ defined by

$$g(x) = \begin{cases} \sin(x) & \text{for } 0 < x < 2\pi, \\ 0 & \text{for } 2\pi < x < 4\pi. \end{cases}$$

3 **a)** For the function $f(x) = \sin(x)$ defined on the half-period $0 < x < \pi$ give the even extension to the full period and compute its Fourier Cosine series.

b) For the function $f(x) = 1 - x$ defined on the half-period $0 < x < 2$ give the odd extension to the full period and compute its Fourier Sine series.

4 Find a particular solution of the ODE

$$y'' + 2y = r(t)$$

where $r(t)$ is the 2-periodic function given by

$$r(t) = \begin{cases} 1 & \text{for } 0 < x < 1, \\ 0 & \text{for } 1 < x < 2. \end{cases}$$