TMA 4135
Matematikk 4D
Høsten 2009

Løsningsforslag – Øving 9

$\boxed{1}$ a) Vi setter $f(x) = u(x,0)$ og får

$$u(x,t) = \frac{1}{2c\sqrt{\pi t}} \int_{-1}^{1} e^{-\frac{(u-x)^2}{4c^2 t}} \, du$$

b) $$u(x,t) = \frac{1}{2c\sqrt{\pi t}} \int_{0}^{\infty} e^{-\frac{(u-x)^2}{4c^2 t}} \, du$$

c) Vi bruker variabelskifte $z = \dfrac{u-x}{2c\sqrt{t}}$ $t>0$

Når $u=0$ er $z = \dfrac{-x}{2c\sqrt{t}}$, og altså

$$u(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\frac{x}{2c\sqrt{t}}}^{\infty} e^{-z^2} \, dz$$

Lf. Øv9. fortsatt

$\boxed{2}$ $x^2 - 20x + 1$

$x = 10 \pm \sqrt{99}$

$\sqrt{99} = 9.94987$ S6

$x_1 = 19.9499$ S6

$x_2 = 0.0101000 = 10 - \sqrt{99}$ S6

$x_2 = 0.100504 = \dfrac{1}{x_1}$ S6

Kommentar: Vi ser at vi mistet 3 signifikante siffer ved å subtrahere 2 nesten like tall.

$\boxed{3}$ Vi har

$x = \dfrac{0.36443}{17.862 - 17.798} = 5.6942$ S5

$x = \dfrac{0.3644}{17.86 - 17.80} = 6.073$ S4

$x = \dfrac{0.364}{17.9 - 17.8} = 3.64$ S3

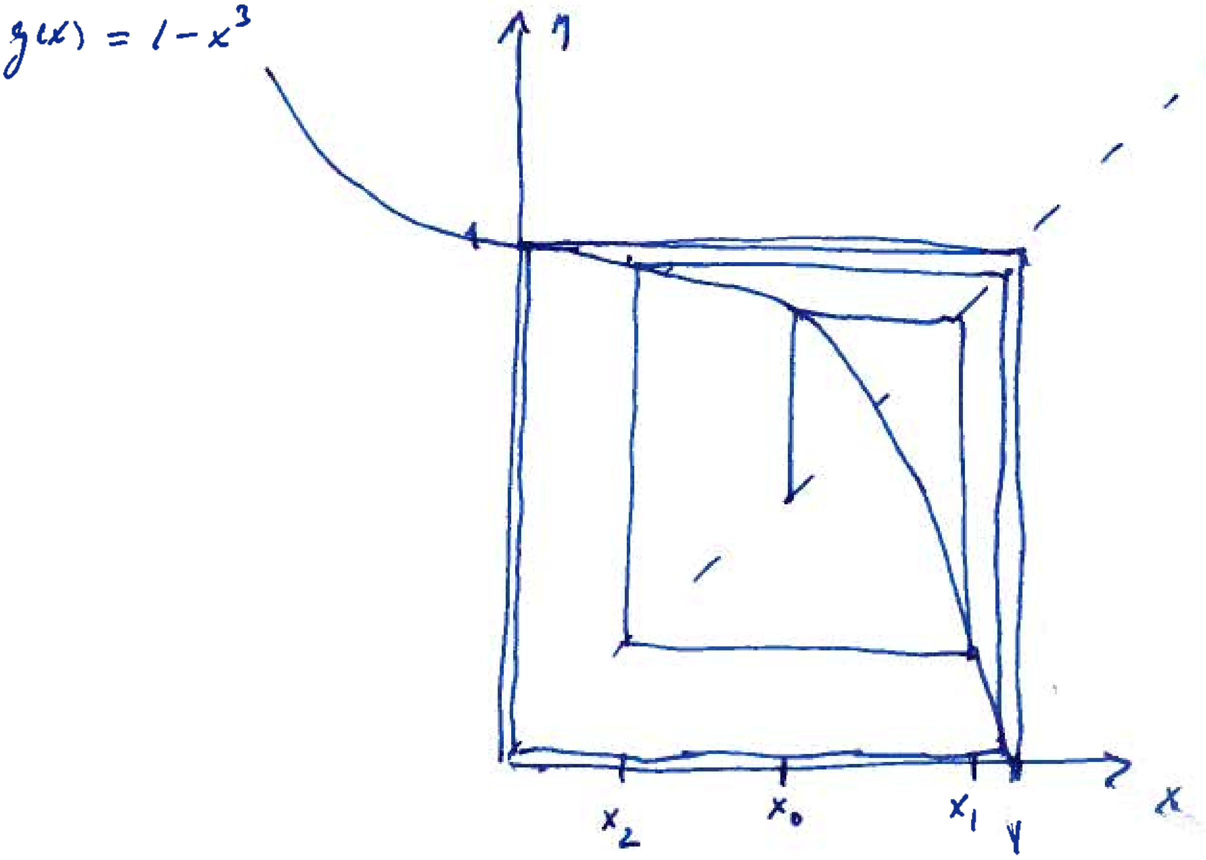
$x = \dfrac{0.36}{18 - 18} = \infty$ S2

$\boxed{4}$

$$g(x) = 1 - x^3$$

Startverdi x_0 $\qquad$ $x_{n+1} = g(x_n)$.

x_0	x_1	x_2	x_3	x_4	x_5	x_6
1	0	1	0	1	0	1
0.5	0.875	0.3300...	0.9646...	0.020945...	0.999990811...	≈ 0
2	−7	344	−40707583	$\approx +\infty$	$\approx -\infty$	$\approx +\infty$



$\boxed{5}$

$$f(x) = x^2 + e^x - 1 = 0$$

$$f'(x) = 2x + e^x$$

$$g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^2 + e^x - 1}{2x + e^x}$$

$$x_0 = -1$$

$$x_1 = g(x_0) = -0.7746003266\ldots$$

$$x_2 = g(x_1) = -0.7186480118\ldots$$

$$x_3 = g(x_2) = -0.7145783107\ldots$$

$$x_4 = g(x_3) = -0.7145563848\ldots$$

$$x_5 = g(x_4) = -0.7145563848\ldots$$

(HP15C)

Residualet for x_3 er $f(x_3)$

$$f(x_3) \approx 0.0000206045\ldots$$

Siden den deriverte i x_3 er ≈ -0.9 er feilen omtrent lik residualet, så 4 siffer er korrekte.
