18–27 IVPs, SOME WITH DISCONTINUOUS INPUT

Using the Laplace transform and showing the details, solve

18. $4y'' - 12y' + 9y = 0, \quad y(0) = 2/3, y'(0) = 1$

19. $y'' - 6y' + 8y = e^{-t} - e^{-4t}, \quad y(0) = 1, y'(0) = 4$

20. $y'' + 10y' + 24y = 144t^2, \quad y(0) = 19/12, y'(0) = -5$

21. $y'' + 4y = 4 \cos t$, if $0 < t < \pi$, and 0 if $t > \pi$

22. $y'' + 3y' + 2y = 4t$ if $0 < t < 1$ and 8 if $t > 1$; $y(0) = 0, y'(0) = 0$

23. $y'' + y' - 2y = 3 \sin t - \cos t$, $(0 < t < 2\pi)$, and $3 \sin 2t - \cos 2t$, $(t > 2\pi)$; $y(0) = 0, y'(0) = -1$

24. $y'' + 3y' + 2y = 1$ if $0 < t < 1$ and 0 if $t > 1$; $y(0) = 0, y'(0) = 0$

25. $y'' + y = 2t$ if $0 < t < 1$ and 2 if $t > 1$

26. Shifted data. $y'' + 2y' + 5y = 10 \sin t$ if $0 < t < 2\pi$ and 0 if $t > 2\pi$; $y(\pi) = 1, y'(\pi) = 2e^{-\pi} - 2$

27. Shifted data. $y'' + 4y = 8t^2$ if $0 < t < 5$ and 0 if $t > 5$; $y(1) = 1 + \cos 2, y'(1) = 4 - 2 \sin 2$

28–40 MODELS OF ELECTRIC CIRCUITS

28–30 *RL*-CIRCUIT

Using the Laplace transform and showing the details, find the current $i(t)$ in the circuit in Fig. 126, assuming $i(0) = 0$ and:

28. $R = 1$ kΩ (=1000 Ω), $L = 1$ H, $v = 0$ if $0 < t < \pi$, and 40 $\sin t$ V if $t > \pi$

29. $R = 25$ Ω, $L = 0.1$ H, $v = 490\, e^{-5t}$ V if $0 < t < 1$ and 0 if $t > 1$

30. $R = 10$ Ω, $L = 0.5$ H, $v = 200t$ V if $0 < t < 2$ and 0 if $t > 2$

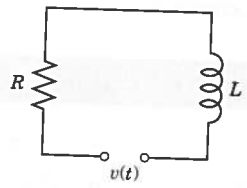

Fig. 126. Problems 28–30

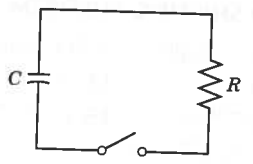

Fig. 127. Problem 31

31. Discharge in *RC*-circuit. Using the Laplace transform, find the charge $q(t)$ on the capacitor of capacitance C in Fig. 127 if the capacitor is charged so that its potential is V_0 and the switch is closed at $t = 0$.

32–34 *RC*-CIRCUIT

Using the Laplace transform and showing the details, find the current $i(t)$ in the circuit in Fig. 128 with $R = 10$ Ω and $C = 10^{-2}$ F, where the current at $t = 0$ is assumed to be zero, and:

32. $v = 0$ if $t < 4$ and $14 \cdot 10^6 e^{-3t}$ V if $t > 4$

33. $v = 0$ if $t < 2$ and $100(t - 2)$ V if $t > 2$

34. $v(t) = 100$ V if $0.5 < t < 0.6$ and 0 otherwise. Why does $i(t)$ have jumps?

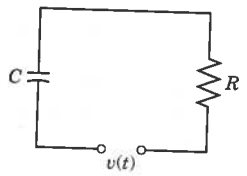

Fig. 128. Problems 32–34

35–37 *LC*-CIRCUIT

Using the Laplace transform and showing the details, find the current $i(t)$ in the circuit in Fig. 129, assuming zero initial current and charge on the capacitor and:

35. $L = 1$ H, $C = 10^{-2}$ F, $v = -9900 \cos t$ V if $\pi < t < 3\pi$ and 0 otherwise

36. $L = 1$ H, $C = 0.25$ F, $v = 200\,(t - \frac{1}{3}t^3)$ V if $0 < t < 1$ and 0 if $t > 1$

37. $L = 0.5$ H, $C = 0.05$ F, $v = 78 \sin t$ V if $0 < t < \pi$ and 0 if $t > \pi$

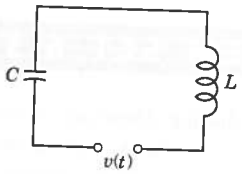

Fig. 129. Problems 35–37

38–40 *RLC*-CIRCUIT

Using the Laplace transform and showing the details, find the current $i(t)$ in the circuit in Fig. 130, assuming zero initial current and charge and:

38. $R = 4$ Ω, $L = 1$ H, $C = 0.05$ F, $v = 34e^{-t}$ V if $0 < t < 4$ and 0 if $t > 4$