	$(a^2 \neq b^2)$	$\frac{1}{2\omega}\sin \omega t$ $\frac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$ $\frac{1}{b^2 - a^2}(\cos at - \cos bt)$	6.6
		$\frac{1}{4k^3}(\sin kt \cos kt - \cos kt \sinh kt)$ $\frac{1}{2k^2}\sin kt \sinh kt$ $\frac{1}{2k^3}(\sinh kt - \sin kt)$ $\frac{1}{2k^2}(\cosh kt - \cos kt)$	
		$\frac{1}{2\sqrt{\pi t^3}}(e^{bt} - e^{at})$ $e^{-(a+b)t/2} I_0\left(\frac{a-b}{2}t\right)$ $J_0(at)$	I 5.5 J 5.4
	0)	$\frac{1}{\sqrt{\pi t}} e^{at}(1 + 2at)$ $\frac{\sqrt{\pi}}{\Gamma(k)}\left(\frac{t}{2a}\right)^{k-1/2} I_{k-1/2}(at)$	I 5.5
		$u(t-a)$ $\delta(t-a)$	6.3 6.4
		$J_0(2\sqrt{kt})$ $\frac{1}{\sqrt{\pi t}}\cos 2\sqrt{kt}$ $\frac{1}{\sqrt{\pi k}}\sinh 2\sqrt{kt}$ $\frac{k}{2\sqrt{\pi t^3}} e^{-k^2/4t}$	J 5.4

(continued)

41	$\ln \frac{s-a}{s-b}$	$\frac{1}{t}(e^{bt} - e^{at})$	
42	$\ln \frac{s^2+\omega^2}{s^2}$	$\frac{2}{t}(1 - \cos \omega t)$	6.6
43	$\ln \frac{s^2-a^2}{s^2}$	$\frac{2}{t}(1 - \cosh at)$	
44	$\arctan \frac{\omega}{s}$	$\frac{1}{t}\sin \omega t$	
45	$\frac{1}{s}\,\mathrm{arccot}\, s$	Si(t)	App. A3.1

CHAPTER 6 REVIEW QUESTIONS AND PROBLEMS

1. State the Laplace transforms of a few simple functions from memory.
2. What are the steps of solving an ODE by the Laplace transform?
3. In what cases of solving ODEs is the present method preferable to that in Chap. 2?
4. What property of the Laplace transform is crucial in solving ODEs?
5. Is $\mathcal{L}\{f(t) + g(t)\} = \mathcal{L}\{f(t)\} + \mathcal{L}\{g(t)\}$? $\mathcal{L}\{f(t)g(t)\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$? Explain.
6. When and how do you use the unit step function and Dirac's delta?
7. If you know $f(t) = \mathcal{L}^{-1}\{F(s)\}$, how would you find $\mathcal{L}^{-1}\{F(s)/s^2\}$?
8. Explain the use of the two shifting theorems from memory.
9. Can a discontinuous function have a Laplace transform? Give reason.
10. If two different continuous functions have transforms, the latter are different. Why is this practically important?

11–19 LAPLACE TRANSFORMS

Find the transform, indicating the method used and showing the details.

11. $3 \cosh t - 5 \sinh 2t$
12. $e^{-2t}(\cos 2t - 4 \sin 2t)$
13. $\cos^2(\frac{1}{2}\pi t)$
14. $16t^2 u(t - \frac{1}{4})$
15. $e^{-t/2}u(t - 2)$
16. $u(t - 2\pi)\cos 2t$
17. $\cos t - t \sin t$
18. $(\sin \omega t) * (\cos \omega t)$
19. $4t * e^{-2t}$

20–28 INVERSE LAPLACE TRANSFORM

Find the inverse transform, indicating the method used and showing the details:

20. $\frac{7.5}{s^2 - 2s - 8}$
21. $\frac{s-1}{s^2}e^{-s}$
22. $\frac{\frac{1}{16}}{s^2 + s + \frac{1}{2}}$
23. $\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$
24. $\frac{s^2 - 6.25}{(s^2 + 6.25)^2}$
25. $\frac{2(1-s)}{s^3}$
26. $\frac{2s - 10}{s^3}e^{-5s}$
27. $\frac{2s + 1}{s^2 + 2s + 5}$
28. $\frac{3s}{s^2 - 2s + 2}$

29–37 ODEs AND SYSTEMS

Solve by the Laplace transform, showing the details and graphing the solution:

29. $y'' + 2y' + 5y = 25t, \quad y(0) = -2, \quad y'(0) = -5$
30. $y'' + 16y = 4\delta(t - \pi), \quad y(0) = -1, \quad y'(0) = 0$