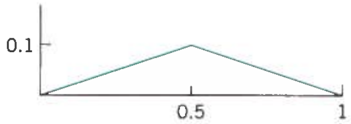
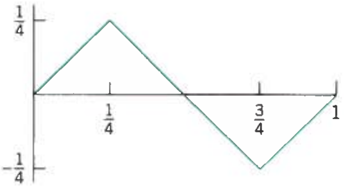
7. $kx(1 - x)$ **8.** $kx^2(1 - x)$

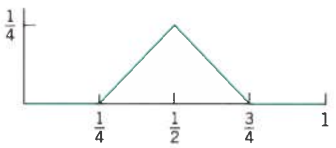
9.



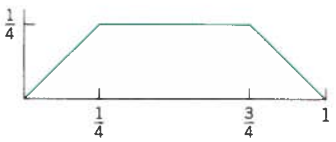
10.



11.



12.



13. $2x - 4x^2$ if $0 < x < \frac{1}{2}$, 0 if $\frac{1}{2} < x < 1$

14. Nonzero initial velocity. Find the deflection $u(x, t)$ of the string of length $L = \pi$ and $c^2 = 1$ for zero initial displacement and "triangular" initial velocity $u_t(x, 0) = 0.01x$ if $0 \leqq x \leqq \frac{1}{2}\pi$, $u_t(x, 0) = 0.01(\pi - x)$ if $\frac{1}{2}\pi \leqq x \leqq \pi$. (Initial conditions with $u_t(x, 0) \neq 0$ are hard to realize experimentally.)

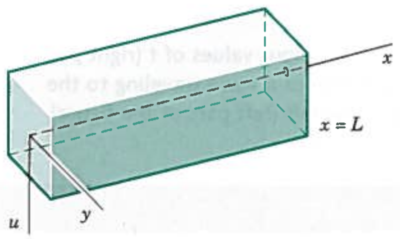


Fig. 292. Elastic beam

15–20 SEPARATION OF A FOURTH-ORDER PDE. VIBRATING BEAM

By the principles used in modeling the string it can be shown that small free vertical vibrations of a uniform elastic beam (Fig. 292) are modeled by the fourth-order PDE

$$\frac{\partial^2 u}{\partial t^2} = -c^2 \frac{\partial^4 u}{\partial x^4} \qquad (21) \qquad \text{(Ref. [C11])}$$

where $c^2 = EI/\rho A$ (E = Young's modulus of elasticity, I = moment of intertia of the cross section with respect to the y-axis in the figure, ρ = density, A = cross-sectional area). (*Bending* of a beam under a load is discussed in Sec. 3.3.)

15. Substituting $u = F(x)G(t)$ into (21), show that

$$F^{(4)}/F = -\ddot{G}/c^2 G = \beta^4 = \text{const},$$
$$F(x) = A \cos \beta x + B \sin \beta x + C \cosh \beta x + D \sinh \beta x,$$
$$G(t) = a \cos c\beta^2 t + b \sin c\beta^2 t.$$

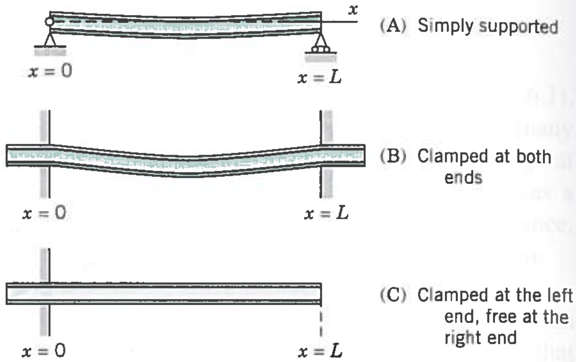


Fig. 293. Supports of a beam

16. Simply supported beam in Fig. 293A. Find solutions $u_n = F_n(x)G_n(t)$ of (21) corresponding to zero initial velocity and satisfying the boundary conditions (see Fig. 293A)

$$u(0, t) = 0, \; u(L, t) = 0$$
(ends simply supported for all times t),
$$u_{xx}(0, t) = 0, \; u_{xx}(L, t) = 0$$
(zero moments, hence zero curvature, at the ends).

17. Find the solution of (21) that satisfies the conditions in Prob. 16 as well as the initial condition

$$u(x, 0) = f(x) = x(L - x).$$

18. Compare the results of Probs. 17 and 7. What is the basic difference between the frequencies of the normal modes of the vibrating string and the vibrating beam?

19. Clamped beam in Fig. 293B. What are the boundary conditions for the clamped beam in Fig. 293B? Show that F in Prob. 15 satisfies these conditions if βL is a solution of the equation

$$\cosh \beta L \cos \beta L = 1. \qquad (22)$$

Determine approximate solutions of (22), for instance, graphically from the intersections of the curves of $\cos \beta L$ and $1/\cosh \beta L$.