The **inverse** of a nonsingular square matrix **A** may be determined in principle by solving the n systems

$$\mathbf{A}\mathbf{x} = \mathbf{b}_j \qquad (j = 1, \cdots, n) \tag{7}$$

where $\mathbf{b}_j$ is the jth column of the $n \times n$ unit matrix.

However, it is preferable to produce $\mathbf{A}^{-1}$ by operating on the unit matrix **I** in the same way as the Gauss–Jordan algorithm, reducing **A** to **I**. A typical illustrative example of this method is given in Sec. 7.8.

PROBLEM SET 20.2

1–5 DOOLITTLE'S METHOD

Show the factorization and solve by Doolittle's method.

1. $4x_1 + 5x_2 = 14$

$12x_1 + 14x_2 = 36$

2. $2x_1 + 9x_2 = 82$

$3x_1 - 5x_2 = -62$

3. $5x_1 + 4x_2 + x_3 = 6.8$

$10x_1 + 9x_2 + 4x_3 = 17.6$

$10x_1 + 13x_2 + 15x_3 = 38.4$

4. $2x_1 + x_2 + 2x_3 = 0$

$-2x_1 + 2x_2 + x_3 = 0$

$x_1 + 2x_2 - 2x_3 = 18$

5. $3x_1 + 9x_2 + 6x_3 = 4.6$

$18x_1 + 48x_2 + 39x_3 = 27.2$

$9x_1 - 27x_2 + 42x_3 = 9.0$

6. TEAM PROJECT. Crout's method factorizes $\mathbf{A} = \mathbf{LU}$, where **L** is lower triangular and **U** is upper triangular with diagonal entries $u_{jj} = 1, j = 1, \cdots, n$.

(a) Formulas. Obtain formulas for Crout's method similar to (4).

(b) Examples. Solve Prob. 5 by Crout's method.

(c) Factor the following matrix by the Doolittle, Crout, and Cholesky methods.

$$\begin{bmatrix} 1 & -4 & 2 \\ -4 & 25 & 4 \\ 2 & 4 & 24 \end{bmatrix}$$

(d) Give the formulas for factoring a tridiagonal matrix by Crout's method.

(e) When can you obtain Crout's factorization from Doolittle's by transposition?

7–12 CHOLESKY'S METHOD

Show the factorization and solve.

7. $9x_1 + 6x_2 + 12x_3 = 17.4$

$6x_1 + 13x_2 + 11x_3 = 23.6$

$12x_1 + 11x_2 + 26x_3 = 30.8$

8. $4x_1 + 6x_2 + 8x_3 = 0$

$6x_1 + 34x_2 + 52x_3 = -160$

$8x_1 + 52x_2 + 129x_3 = -452$

9. $0.01x_1 + 0.03x_3 = 0.14$

$0.16x_2 + 0.08x_3 = 0.16$

$0.03x_1 + 0.08x_2 + 0.14x_3 = 0.54$

10. $4x_1 + 2x_3 = 1.5$

$4x_2 + x_3 = 4.0$

$2x_1 + x_2 + 2x_3 = 2.5$

11. $x_1 - x_2 + 3x_3 + 2x_4 = 15$

$-x_1 + 5x_2 - 5x_3 - 2x_4 = -35$

$3x_1 - 5x_2 + 19x_3 + 3x_4 = 94$

$2x_1 - 2x_2 + 3x_3 + 21x_4 = 1$

12. $4x_1 + 2x_2 + 4x_3 = 20$

$2x_1 + 2x_2 + 3x_3 + 2x_4 = 36$

$4x_1 + 3x_2 + 6x_3 + 3x_4 = 60$

$2x_2 + 3x_3 + 9x_4 = 122$

13. Definiteness. Let **A**, **B** be $n \times n$ and positive definite. Are $-\mathbf{A}$, $\mathbf{A}^{\mathsf{T}}$, $\mathbf{A} + \mathbf{B}$, $\mathbf{A} - \mathbf{B}$ positive definite?