sum of this inverse and (7) is the solution of the problem for $0 < t < \pi$, namely (the sines cancel),

$$y(t) = 3e^{-t}\cos t - 2\cos 2t - \sin 2t \qquad \text{if } 0 < t < \pi.$$

e second fraction in (6), taken with the minus sign, we have the factor $e^{-\pi s}$, so that from (8) and the second ing theorem (Sec. 6.3) we get the inverse transform of this fraction for $t > 0$ in the form

$$+2\cos(2t - 2\pi) + \sin(2t - 2\pi) - e^{-(t-\pi)}[2\cos(t - \pi) + 4\sin(t - \pi)]$$
$$= 2\cos 2t + \sin 2t + e^{-(t-\pi)}(2\cos t + 4\sin t).$$

sum of this and (9) is the solution for $t > \pi$,

$$y(t) = e^{-t}[(3 + 2e^{\pi})\cos t + 4e^{\pi}\sin t] \qquad \text{if } t > \pi.$$

re 136 shows (9) (for $0 < t < \pi$) and (10) (for $t > \pi$), a beginning vibration, which goes to zero rapidly use of the damping and the absence of a driving force after $t = \pi$. ■

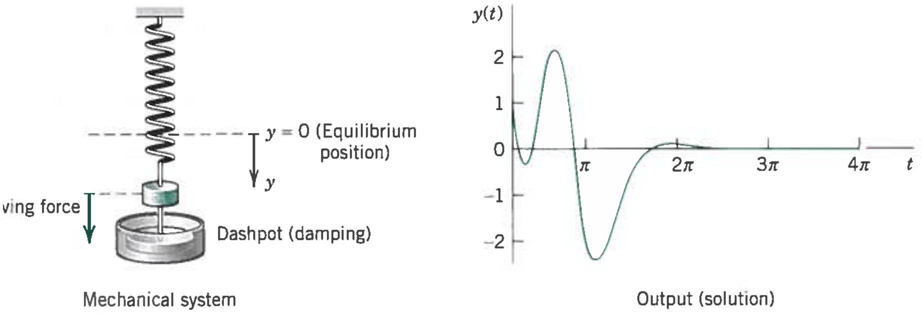

Fig. 136. Example 4

: case of repeated complex factors $[(s - a)(s - \bar{a})]^2$, which is important in connection h resonance, will be handled by "convolution" in the next section.

ET 6.4

fect of Damping. Consider a ur choice modeled by

$\cdot y' + ky = \delta(t)$.

e solution, describe the effect of ıg the damping to 0, keeping k

c is kept constant and k is l, starting from 0?

ults to a system with two ıt, acting at different times.

. Limit of a Rectangular Wave.

ıe text, take a rectangular wave $+ k$. Graph the responses for a k approaching zero, illustrating maller k those curves approach

the curve shown in Fig. 134. *Hint:* If your CAS gives no solution for the differential equation, involving k, take specific k's from the beginning.

(b) Experiment on the response of the ODE in Example 1 (or of another ODE of your choice) to an impulse $\delta(t - a)$ for various systematically chosen a (> 0); choose initial conditions $y(0) \neq 0$, $y'(0) = 0$. Also consider the solution if no impulse is applied. Is there a dependence of the response on a? On b if you choose $b\delta(t - a)$? Would $-\delta(t - \tilde{a})$ with $\tilde{a} > a$ annihilate the effect of $\delta(t - a)$? Can you think of other questions that one could consider experimentally by inspecting graphs?

3–12 EFFECT OF DELTA (IMPULSE) ON VIBRATING SYSTEMS

Find and graph or sketch the solution of the IVP. Show the details.

3. $y'' + 9y = \delta(t - \pi/2), \quad y(0) = 2, y'(0) = 0$

4. $y'' + 16y = 4\delta$
5. $y'' + 4y = \delta(t$
$y(0) = 0, y'(0)$
6. $y'' + 4y' + 5y$
7. $4y'' + 16y' +$
$y(0) = \frac{3}{5}, y'(0)$
8. $y'' + 3y' + 2y$
$y'(0) = -1$
9. $y'' + 2y' + 2$
$y(0) = 0, y'(0$
10. $y'' + 5y' + 6$
$y(0) = 0, y'(0$
11. $y'' + 3y' + 2$
$y(0) = 0, y'(0$
12. $y'' + 2y' + 5$
$y'(0) = 5$
13. PROJECT. I
a simple root
have the *Hea*

(b) Similarly
fractions in

$\frac{F(s)}{G(s)} =$

+

we have the

and for the

$A_k =$

14. TEAM PR
Functions
(a) Theor
continuous

(11)

Prove this