14–20 INVERSE TRANSFORMS

Using differentiation, integration, s-shifting, or convolution, and showing the details, find $f(t)$ if $\mathcal{L}(f)$ equals:

14. $\dfrac{s}{(s^2+16)^2}$

15. $\dfrac{s}{(s^2-4)^2}$

16. $\dfrac{2s+6}{(s^2+6s+10)^2}$

17. $\ln \dfrac{s}{s-1}$

18. $\operatorname{arccot} \dfrac{s}{\pi}$

19. $\ln \dfrac{s^2+1}{(s-1)^2}$

20. $\ln \dfrac{s+a}{s+b}$

6.7 Systems of ODEs

The Laplace transform method may also be used for solving systems of ODEs, as we shall explain in terms of typical applications. We consider a first-order linear system with constant coefficients (as discussed in Sec. 4.1)

$$(1) \qquad \begin{aligned} y_1' &= a_{11}y_1 + a_{12}y_2 + g_1(t) \\ y_2' &= a_{21}y_1 + a_{22}y_2 + g_2(t). \end{aligned}$$

Writing $Y_1 = \mathcal{L}(y_1)$, $Y_2 = \mathcal{L}(y_2)$, $G_1 = \mathcal{L}(g_1)$, $G_2 = \mathcal{L}(g_2)$, we obtain from (1) in Sec. 6.2 the subsidiary system

$$\begin{aligned} sY_1 - y_1(0) &= a_{11}Y_1 + a_{12}Y_2 + G_1(s) \\ sY_2 - y_2(0) &= a_{21}Y_1 + a_{22}Y_2 + G_2(s). \end{aligned}$$

By collecting the Y_1- and Y_2-terms we have

$$(2) \qquad \begin{aligned} (a_{11} - s)Y_1 + \quad a_{12}Y_2 \quad &= -y_1(0) - G_1(s) \\ a_{21}Y_1 \quad + (a_{22} - s)Y_2 &= -y_2(0) - G_2(s). \end{aligned}$$

By solving this system algebraically for $Y_1(s), Y_2(s)$ and taking the inverse transform we obtain the solution $y_1 = \mathcal{L}^{-1}(Y_1)$, $y_2 = \mathcal{L}^{-1}(Y_2)$ of the given system (1).

Note that (1) and (2) may be written in vector form (and similarly for the systems in the examples); thus, setting $\mathbf{y} = [y_1 \quad y_2]^\mathsf{T}$, $\mathbf{A} = [a_{jk}]$, $\mathbf{g} = [g_1 \quad g_2]^\mathsf{T}$, $\mathbf{Y} = [Y_1 \quad Y_2]^\mathsf{T}$, $\mathbf{G} = [G_1 \quad G_2]^\mathsf{T}$ we have

$$\mathbf{y}' = \mathbf{A}\mathbf{y} + \mathbf{g} \qquad \text{and} \qquad (\mathbf{A} - s\mathbf{I})\mathbf{Y} = -\mathbf{y}(0) - \mathbf{G}.$$

EXAMPLE 1 Mixing Problem Involving Two Tanks

Solution. The model is obtained in the form of t

Time rate of change =

for the two tanks (see Sec. 4.1). Thus,

$$y_1' = -\tfrac{8}{100}y_1 + \tfrac{2}{100}y_2 +$$

The initial conditions are $y_1(0) = 0$, $y_2(0) = 150$. Fr

$$(-0.08 - s)Y_1 +$$

$$0.08Y_1 \qquad +$$

We solve this algebraically for Y_1 and Y_2 by elimin solutions in terms of partial fractions,

$$Y_1 = \frac{9s + 0.48}{s(s+0.12)(s+0.04)}$$

$$Y_2 = \frac{150s^2 + 12s + 0.48}{s(s+0.12)(s+0.04)}$$

By taking the inverse transform we arrive at the solu

$$y_1 = 100 - 62.5$$

$$y_2 = 100 + 125$$

Figure 144 shows the interesting plot of these func features? Why do they have the limit 100? Why is y_2 on suddenly larger than y_2? Etc.

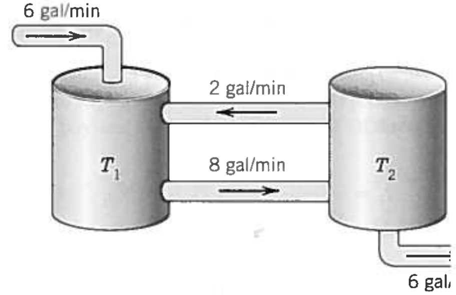

Fig. 144. Mixing

Other systems of ODEs of practical impo method in a similar way, and eigenvalues in Chap. 4, will come out automatically, a

EXAMPLE 2 Electrical Network

Find the currents $i_1(t)$ and $i_2(t)$ in the network in Fi