In the back substitution of x_i we make $n - i$ multiplications and as many subtractions, as well as 1 division. Hence the number of operations in the back substitution is

$$b(n) = 2\sum_{i=1}^{n} (n - i) + n = 2\sum_{s=1}^{n} s + n = n(n + 1) + n = n^2 + 2n = O(n^2).$$

We see that it grows more slowly than the number of operations in the forward elimination of the Gauss algorithm, so that it is negligible for large systems because it is smaller by a factor n, approximately. For instance, if an operation takes 10^{-9} sec, then the times needed are:

Algorithm	$n = 1000$	$n = 10000$
Elimination	0.7 sec	11 min
Back substitution	0.001 sec	0.1 sec

PROBLEM SET 20.1

APPLICATIONS of linear systems see Secs. 7.1 and 8.2.

1–3 GEOMETRIC INTERPRETATION

Solve graphically and explain geometrically.

1. $x_1 - 4x_2 = 20.1$

$3x_1 + 5x_2 = 5.9$

2. $-5.00x_1 + 8.40x_2 = 0$

$10.25x_1 - 17.22x_2 = 0$

3. $7.2x_1 - 3.5x_2 = 16.0$

$-14.4x_1 + 7.0x_2 = 31.0$

4–16 GAUSS ELIMINATION

Solve the following linear systems by Gauss elimination, with partial pivoting if necessary (but without scaling). Show the intermediate steps. Check the result by substitution. If no solution or more than one solution exists, give a reason.

4. $6x_1 + x_2 = -3$

$4x_1 - 2x_2 = 6$

5. $2x_1 - 8x_2 = -4$

$3x_1 + x_2 = 7$

6. $25.38x_1 - 15.48x_2 = 30.60$

$-14.10x_1 + 8.60x_2 = -17.00$

7. $-3x_1 + 6x_2 - 9x_3 = -46.725$

$x_1 - 4x_2 + 3x_3 = 19.571$

$2x_1 + 5x_2 - 7x_3 = -20.073$

8. $5x_1 + 3x_2 + x_3 = 2$

$-4x_2 + 8x_3 = -3$

$10x_1 - 6x_2 + 26x_3 = 0$

9. $6x_2 + 13x_3 = 137.86$

$6x_1 - 8x_3 = -85.88$

$13x_1 - 8x_2 = 178.54$

10. $4x_1 + 4x_2 + 2x_3 = 0$

$3x_1 - x_2 + 2x_3 = 0$

$3x_1 + 7x_2 + x_3 = 0$

11. $3.4x_1 - 6.12x_2 - 2.72x_3 = 0$

$-x_1 + 1.80x_2 + 0.80x_3 = 0$

$2.7x_1 - 4.86x_2 + 2.16x_3 = 0$

12. $5x_1 + 3x_2 + x_3 = 2$

$-4x_2 + 8x_3 = -3$

$10x_1 - 6x_2 + 26x_3 = 0$