25–27 HEAT

Find the temperature distribution in a laterally insulated thin copper bar ($c^2 = K/(\sigma\rho) = 1.158\ \text{cm}^2/\text{sec}$) of length 100 cm and constant cross section with endpoints at $x = 0$ and 100 kept at 0°C and initial temperature:

25. $\sin 0.01\pi x$

26. $50 - |50 - x|$

27. $\sin^3 0.01\pi x$

28–30 ADIABATIC CONDITIONS

Find the temperature distribution in a laterally insulated bar of length π with $c^2 = 1$ for the adiabatic boundary condition (see Problem Set 12.6) and initial temperature:

28. $3x^2$

29. $100 \cos 2x$

30. $2\pi - 4|x - \frac{1}{2}\pi|$

31–32 TEMPERATURE IN A PLATE

31. Let $f(x, y) = u(x, y, 0)$ be the initial temperature in a thin square plate of side π with edges kept at 0°C and faces perfectly insulated. Separating variables, obtain from $u_t = c^2\nabla^2 u$ the solution

$$u(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \sin mx \sin ny\, e^{-c^2(m^2+n^2)t}$$

where

$$B_{mn} = \frac{4}{\pi^2}\int_0^{\pi}\int_0^{\pi} f(x, y) \sin mx \sin ny\, dx\, dy.$$

32. Find the temperature in Prob. 31 if $f(x,y) = x(\pi - x)y(\pi - y)$.

33–37 MEMBRANES

Show that the following membranes of area 1 with $c^2 = 1$ have the frequencies of the fundamental mode as given (4-decimal values). Compare.

33. Circle: $\alpha_1/(2\sqrt{\pi}) = 0.6784$

34. Square: $1/\sqrt{2} = 0.7071$

35. Rectangle with sides $1{:}2{:}\sqrt{5/8} = 0.7906$

36. Semicircle: $3.832/\sqrt{8\pi} = 0.7643$

37. **Quadrant** of circle: $\alpha_{21}/(4\sqrt{\pi}) = 0.7244$ ($\alpha_{21} = 5.13562$ = first positive zero of J_2)

38–40 ELECTROSTATIC POTENTIAL

Find the potential in the following charge-free regions.

38. Between two concentric spheres of radii r_0 and r_1 kept at potentials u_0 and u_1, respectively.

39. Between two coaxial circular cylinders of radii r_0 and r_1 kept at the potentials u_0 and u_1, respectively. Compare with Prob. 38.

40. In the interior of a sphere of radius 1 kept at the potential $f(\phi) = \cos 3\phi + 3\cos\phi$ (referred to our usual spherical coordinates).

SUMMARY OF CHAPTER 12
Partial Differential Equations (PDEs)

Whereas ODEs (Chaps. 1–6) serve as models of problems involving only *one* independent variable, problems involving *two or more* independent variables (space variables or time t and one or several space variables) lead to PDEs. This accounts for the enormous importance of PDEs to the engineer and physicist. Most important are:

(1) $u_{tt} = c^2 u_{xx}$ One-dimensional wave equation (Secs. 12.2–12.4)

(2) $u_{tt} = c^2(u_{xx} + u_{yy})$ Two-dimensional wave equation (Secs. 12.8–12.10)

(3) $u_t = c^2 u_{xx}$ One-dimensional heat equation (Secs. 12.5, 12.6, 12.7)

(4) $\nabla^2 u = u_{xx} + u_{yy} = 0$ Two-dimensional Laplace equation (Secs. 12.6, 12.10)

(5) $\nabla^2 u = u_{xx} + u_{yy} + u_{zz} = 0$ Three-dimensional Laplace equation (Sec. 12.11).

Equations (1) and (2) are hyperbolic, (3) is parabolic, (4) and (5) are elliptic.