Norwegian University of Science and Technology
Department of Mathematics

TMA4145 Linear Methods
Autumn 2012

Exercise set 1

1 A deck of cards can be considered a set, where each element (i.e. card) has a *value* and a *suit*. Given an element x, x_v is its value and x_s its suit (if x is the Queen of hearts, $x_v = 12$ and $x_s =$ hearts). Let the set D be a standard deck of 52 cards.

a) Find the cardinality of the following sets:

1. $S = \{x \in D | x_s = \text{spades}\}$
2. $B = \{\text{the set of court cards in } D\}$[1]
3. $S \cap B$
4. $S \cup B$
5. $S \setminus B$
6. $S \times B$
7. $\{(x, y) \in D \times D | x = y\}$

b) Fill in the correct symbol: $\in$, $\subseteq$ or $\nsubseteq$

1. ace of hearts $\square D$
2. $\{\text{ace of hearts}\}\square D$
3. $S\square D$
4. $S\square B$

c) Let $A = \{C \subseteq D | C \text{ contains all card of a suit}\}$. Are the following statements true or false?

1. $\forall C \in A\, \exists\, x \in C$ such that $x_v = 5$.
2. $\forall C \in A\, \exists!\, x \in C$ such that $x_v < 5$.
3. $\exists x \in D$ such that $x_v \leq y_v\, \forall\, y \in D$.

2 Consider the sets $A = \{\blacksquare, \triangle, \Diamond\}$ and $B = \{\circ, \star\}$.

a) How many functions $f : A \to B$ exist? (i.e. with domain A and codomain B)

b) How many functions f are there with domain A and *range* B?

c) How many injective functions are there from B to A?

d) Given two arbitrary, finite sets X and Y, how many different functions are there from X to Y?

[1] Court cards: Jack, Queen, King

3 Find a domain A such that the following mappings f are bijections $A \to f(A)$. Try to choose A as large as possible. What is $f(A)$ and f^{-1} for your choice of A?

a) $x \mapsto x^3 \qquad A \subseteq \mathbb{R}$

b) $x \mapsto \sin(x) \qquad A \subseteq \mathbb{R}$

c) $x \mapsto \tan(x) \qquad A \subseteq \mathbb{R}$

d) $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ x+2y \end{bmatrix} \qquad A \subseteq \mathbb{R}^2$

e) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ 0 \\ x+z \end{bmatrix} \qquad A \subseteq \mathbb{R}^3$

4 **a)** Find a bijection $f\colon \mathbb{Z} \to \mathbb{N}$.

b) Challenge: Find a bijection $f\colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$. (Can you think of how to use this bijection to show that the set of rational numbers $\mathbb{Q}$ is *countable*?)