Norwegian University of Science and Technology
Department of Mathematics

TMA4145 Linear Methods Autumn 2012

Exercise set 3

1 *(Strang 2.1.2)* Which of the following subsets of $\mathbb{R}^3$ are vector spaces when we define addition and scalar multiplication as on $\mathbb{R}^3$?

Give a proof or a counterexample in each case.

a) $\{(0, y, z) \in \mathbb{R}^3\}$

b) $\{(1, y, z) \in \mathbb{R}^3\}$

c) $\{(x, y, z) \in \mathbb{R}^3 | yz = 0\}$

d) All linear combinations $\lambda u + \mu v$ where $u = (1, 1, 0)$, $v = (2, 0, 1)$ and $\lambda, \mu \in \mathbb{R}$

e) $\{(x, y, z) \in \mathbb{R}^3 | z - y + 3x = 0\}$

2 *(Kreyzig 1.3.2)* What is an open ball $B_1(x_0)$

a) in $\mathbb{R}$ with metric $d(x, y) = |x - y|$?

b) in $\mathbb{C}$ with the metric induced by the norm $\|x\| = \sqrt{\mathrm{Re}(x)^2 + \mathrm{Im}(x)^2}$?

c) in $BC([a, b], \mathbb{R})$ with the metric induced by the supremum norm?

3 *(Kreyzig 1.3.5)*

a) Let (X, d) be a metric space. Show that X and $\emptyset$ are both open and closed as subsets of X.

b) Show that in a discrete metric space, every subset is both open and closed.

4 *(Kreyzig 1.4.6)* If (x_n) and (y_n) are Cauchy sequences in a metric space (X, d), show that (a_n) where $a_n = d(x_n, y_n)$ converges. Give an example where this happens.

5 *(Kreyzig 1.1.7)* If A is the subspace of l^∞ consisting of all sequences of zeros and ones, what is the induced metric on A?

6 *Challenge*

a) *(Kreyzig 1.2.4)* Find a sequence that converges to 0, but which is not in any space l^p where $1 \leq p < \infty$.

b) *(Kreyzig 1.2.5)* Find a sequence x such that $x \notin l^1$, but $x \in l^p$ for $p > 1$.