Norwegian University of Science and Technology
Department of Mathematics

TMA4145 Linear Methods
Autumn 2012

Exercise set 6

1 *(Strang 2.3.33)* Find a basis for each of these subspaces of $M_{3\times 3}(\mathbb{R})$:

a) All diagonal matrices ($a_{ij} = 0$ for $i \neq j$).

b) All symmetric matrices ($A^T = A$).

c) All skew-symmetric matrices ($A^T = -A$).

2 *(Strang 1.5.7)*

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}, \qquad b = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$$

a) Show by performing Gaussian elimination that the LU-factorisation of A is given by:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix}$$

b) Write down the upper triangular system $Ux = c$ which appears after Gaussian elimination of $Ax = b$.

3 *(Strang 1.5.29)* The equation $Ax = b$ can be solved via $LUx = b$, by first solving $Lc = b$ for c, and then solving $Ux = c$ for x.

a) Do this for

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

b) Find A.

4 *(Strang 1.6.39)* Invert these matrices by the Gauss-Jordan method starting with $[A\,I]$

a)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \text{ (Answer: } A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix})$$

b)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

5 *(Challenge)*

a) *(Friedberg 1.6.24)* Let f be a polynomial of degree n in $P_n(\mathbb{R})$. Prove that for any $g \in P_n(\mathbb{R})$ there exist scalars $c_0, c_1, \ldots c_n$ such that

$$g(x) = c_0 f(x) + c_1 f'(x) + c_2 f''(x) + \ldots + c_n f^{(n)}(x),$$

where $f^{(n)}$ denotes the n^{th} derivative of f.

b) Let $f(x) = x^3 + x$. According to **a)** the set $F = \{f(x), f'(x), f''(x), f'''(x)\}$ generate $P_3(\mathbb{R})$. Moreover, F is a basis for $P_3(\mathbb{R})$. Find the change-of-basis matrix from the standard basis $E = \{1, x, x^2, x^3\}$ for $P_3(\mathbb{R})$ to the basis F.
Use this to calculate the coordinates (c_0, c_1, c_2, c_3) in **a)** for $g(x) = x^3 - x^2 + 2$.